Model of Diversity: Systematic Analysis and Application to Russian Economy

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- Diversity: groups that are or/and aware of being different;
- More cultural diversity. Two sides:

  - variety of cultures, opportunities for sharing and learning from others;

  - cultural differences: difficult to understand, possible conflicts

Alberto Alesina & Eliana La Ferrara «ETHNIC DIVERSITY AND ECONOMIC PERFORMANCE», 2005

«Conflict of preferences, racism, prejudices often lead to policies which are suboptimal from the point of view of society as a whole, and to the oppression of minorities which may then explode in civil wars or at least in disruptive political instability».

«But an ethnic mix also brings about variety in abilities, experiences, cultures which may be productive and may lead to innovation and creativity. The United States are the quintessential example of these two faces of racial relations in a “melting pot”».

Motivation:

- A simple model comparing collaboration costs and benefits of diversity;
- Endogenous diversity: factors contributing to the optimal diversity.

In a wider sense:

Why diversity indexes are significant in growth regressions (?): considering differences between agents in the model.

Literature

- Trade-off between heterogeneity and size of states (Alesina & Spolaore, 2005, Desmet et. al. 2007)
- Trade-off between productive benefits of diversity and costs of communication (Lazear, 1999 a, b; O'Reilly Williams and Barsade, 1997)
- Double nature of fragmentation (Alesina and La Ferrara, 2004): diversity as a factor of production, alienation acting on preferences
- Distribution of heterogeneous workers in US city markets based on how heterogeneity acts on popular preferences and firms' efficiency Ottaviano and Peri, 2005)
- In Russia: Bufetova, Kolomak & Mikhaleva, 2017; Limonov and Nesena, 2015 (In Russ.) – Panel data: Diversity supports economic outcome in Russia (census statistics)

Base model (Suslov N. Optimal diversity in socio-economic system: formal aspects. Proceedings of IX Moscow international conference on operation research (ORM2018). Volume II, Moscow, October, 22-27, 2018, pp. 229-223)

- Rent seeking activities require spending real resources (Tullock, 1967) that are subtracted from the total volume.
- Production process creates a common result / prize (administrative rent). Or this result / prize may come from the outside.
- Every agent has a limited resource to use in production that generates labor income or to rent out in order to get rent income.
- The agents are grouped homogeneously.
- In the long run, diversity (as a number of homogeneous groups) has a positive impact on productivity.

The number of primary agents – N, the number of groups with homogeneous agents – n, and also $N \geq n$. The number of agents in group i is n_i, and the share of group i in the total number of agents –

$$s_i = \frac{n_i}{N}$$

Every primary agent has a resource of 1, and group i in total – n_i or in relative units s_i. The group maximizes its total income:

$$\max\left\{s_i \cdot a \cdot (1 - x_i) \cdot (1 - t) + \delta_i \cdot t \cdot a \cdot (1 - X)\right\}$$
$$0 \leq x_i \leq 1$$

where t is an institutional variable – a share of rent in the income or a share of income in the 'common prize', x_i - a volume of rent seeking activity of one agent from group i, δ_i - a share of group i in the total rent volume, a is resource productivity, X – the total volume of rent seeking activity:

$$X = \sum_{i=1}^{n} s_i \cdot x_i$$

The share of group *i* in the total rent volume:

$$\delta_i = \frac{s_i \cdot x_i}{\sum_{j=1}^{n} s_j \cdot x_j} = \frac{s_i \cdot x_i}{X}$$

The volume of production activity of one primary agent of group *i*:

$y_i=1-x_i$, the total volume of production activity:

$$Y = \sum_{i=1}^{n} s_i \cdot x_i = 1 - X$$

Thus, $x_i+y_i=1$ and $X+Y=1$

Solution

$$X = \frac{n-1}{n} \cdot t, \qquad Y = \frac{n-(n-1)\cdot t}{n},$$

Diversity optimization

Assuming that all groups have the same size: $s_i = 1/n$

Output $Q = \hat{a}(n) \cdot Y \qquad \hat{a}' > 0, \hat{a}'' < 0, \qquad \hat{a}(\infty) = const$

Let $\hat{a}(n) = \left(b + c \cdot \frac{n-1}{n} \right)^{\frac{\mu}{\mu-1}} = \left(b + c \cdot m \right)^{\frac{\mu}{\mu-1}},$ where $m = \frac{n-1}{n}$

Optimizing diversity

$$\max Q = \max\left[a(m)\cdot Y(m)\right] = \max\left(b + c\cdot m\right)^{\frac{\mu}{\mu-1}}\cdot\left(1 - t\cdot m\right)$$

The maximum product is achieved when:

$$\frac{dQ}{dm} = \gamma\cdot\left(b + c\cdot m\right)^{\frac{1}{\mu-1}}\cdot\left(1 - t\cdot m\right)\cdot c - \left(b + c\cdot m\right)^{\frac{\mu}{\mu-1}}\cdot t = 0$$

$$m^{*} = \frac{n^{*} - 1}{n^{*}} = (1-\mu)\cdot\left(\frac{\mu}{(1-\mu)\cdot t} - \frac{b}{c}\right)$$

μ – propensity to rent seeking activity,

b – productivity growth not connected to a higher number of groups,

c – productivity growth due to a higher number of groups.

Assumption: groups that feel less ‘alienated’ are more likely to socialize and, thus, engage in rent seeking activity.

Alienation index - Desmet K., Ortuno-Ortin I., Weber S. – 2009):

$$A(\alpha, T) = \sum_{i=1}^{n} \sum_{j=1}^{n} s_i^{\alpha+1} \cdot s_j \cdot \tau_{ij}$$

α - a level of ‘group’s self-identification’ that may be positive as well as negative, τ_{ij} – ‘cultural distance’ between members of group i and j.

If $\alpha = 0, \tau_{ij} = 1, \ i \neq j, \tau_{ij} = 0, \ i = j$, then

ELF index= $A(0, \hat{T}) = I_{ELF} = \sum_{i=1}^{n} s_i \cdot (1 - s_i) = 1 - \sum_{i=1}^{n} s_i^2$

Individual alienation level for one member of group *i*:

$$u_i(s_i) = 1 - s_i$$

The problem:

$$\max\left\{s_i \cdot (1 - x_i) \cdot (1 - t) + \sigma_i \cdot t \cdot (1 - X)\right\}$$

$$\sigma_i = \frac{u_i \cdot s_i \cdot x_i}{\sum_{j=1}^{n} u_j \cdot s_j \cdot x_j} = \frac{u_i \cdot s_i \cdot x_i}{Z}$$

FOC: $$\sigma_i = 1 - \frac{1}{(u_i \cdot R + 1) \cdot t}, \text{ where } \frac{Y}{Z} = R$$

Solution

1. $$(n-1)\cdot t=\sum_{i=1}^{n}\frac{1}{u_i\cdot R+1} \quad \Rightarrow \quad R$$

2. $$1-X=R\cdot Z,$$

$$\sum_{i=1}^{n}s_i\cdot x_i=X,$$

$$s_i\cdot x_i=\frac{1}{u_i}\cdot\left(Z-\frac{Z}{(u_i\cdot R+1)\cdot t}\right)$$

Proof: the higher is the ***ELF-index***, the higher is the volume of rent seeking activities ***X.***

Main variables used: year of 2002

Variable	Obs.	Mean	Std. Dev.	Min	Max
Diversity index: ELF	83	0.304	0.196	0.067	0.838
Change of ELF up to 2010	83	1.176	0.305	0.350	2.097
Number of ethnic groups	83	111.29	14.860	66	142

Sources: ROSSTAT, 2002 and 2010 population censuses

- ***Control variables for 2002***: per capita income: GRP, investment climate indices, infant mortality rate, investment share in GRP,
- ***Variables for 2003-2010 period***: GRP growth rate, migration and natural population growth rates.
- ***Variable for 1998***: industry production growth rate.

Estimation of Diversity Change Rate for 2003-2010 in Russian regions. Dependent variable Ln*(ELF-Index Growth Rate). White Estimator of covariance matrix, 79 observations*

Constant	-1.178***	-.878***	-.636***	-.504***	-1.135***	-1.216***
ln(ELF)	-0.621***	-0.603***	-0.560***	-0.533***	-0.546***	-0.493***
ELF*ln(Initial p.c. Income)	0.135***	0.135***	0.133***	0.125***	0.140***	0.125***
Infant mortality		-0.019***				
Investment risk			-0.417***	-0.459***		-0.285***
Investment potential				-0.020***		-0.029***
Migration rate					1.114**	1.051***
Natural population growth rate					-1.888**	-1.216***
R-Squared	.4990	.5911	.6540	.6746	.6535	.7339
Fisher statistics	37.59	24.58	33.37	27.13	22.49	21.82

** and *** - stand to indicate 5 and 1 percent significance levels correspondingly

Estimation of a System of Regressions **1** *for 2003-2010 for Russian regions. Three-stage least squares estimator, 79 observations*

	Ln (GRP growth rate)	Ln (Diversity change rate)
Constant term	-0.238	-0.474**
Diversity index: ELF	2.183**	
ln(Diversity index: ELF)		-0.521***
ELF*ln(Initial p.c. income)	-0.181**	0.121***
Number of ethnic groups	0.005***	
Investment share in GRP in 2002	0.424***	
Industry production in 1998 to 1991	-0.312***	
Investment risk		-0.459***
Investment potential		-0,020**
“R-squared”	0.3045	0.6745
Hi-squared	34.96	162.17

** and *** - stand to indicate 5 and 1 percent significance levels correspondingly

Estimation of a System of Regressions **2** *for 2003-2010 for Russian regions. Three-stage least squares estimator, 79 observations*

	Ln (GRP growth rate)	Ln (Diversity change rate)	Ln (Average inv.-t share)
Constant term	0.704***	-0.504**	-1.779***
ln(Diversity change rate)	-0.156**		
Migration rate	1.442***		
ln(Average inv.-t share)	0.224***		
ln(Diversity index: ELF)		-0.539***	
ELF*ln(Initial p.c. income)		0.126***	
Investment risk		-0.501***	
Ln(Investment potential)		-0.051**	
ln(GRP growth rate)			0.834***
Investment potential			-0.055***
Investment share in 2002			1.047***
“R-squared”	0.3420	0.6707	0.5144
Hi-squared	29.17	165.02	87.02

** and *** - stand to indicate 5 and 1 percent significance levels respectively

Results: coefficients of elasticity with respect to initial levels of diversity: ELF in 2002

Variable	Obs.	Mean	Std. Dev.	Min	Max
Income elasticity	79	0.042	0.056	-0.093	0.267
Diversity change elasticity	79	-0.138	0,247	-0.434	0.462

Results by Federal Districts

	Diversity in 2002	Change of diversity	Diversity elasticity	Income elasticity
Central Federal Okrug	0,116	1,468	-0,370	0,015
North-west Federal Okrug	0,273	1,124	-0,195	0,019
Southern Federal Okrug	0,378	1,066	-0,065	0,071
North-Caucauses Federal Okrug	0,505	0,925	0,159	0,161
Volga Federal Okrug	0,406	1,046	0,000	0,063
Urals Federal Okrug	0,389	1,066	-0,120	0,001
Siberian Federal Okrug	0,278	1,025	-0,166	0,045
Far-East Federal Okrug	0,335	1,034	-0,056	-0,002

Principal conclusions:

- There is an optimal diversity arising from interaction costs and benefits of diversity. Other critical factors include propensity to rent seeking activity and agents' alienation level;
- Between the census of 2002 and that of 2010 in most regions and in Russia as a whole the level of ethnic heterogeneity increased and sustained the economic growth;
- The factors of growing heterogeneity were international and inter-regional migration as well as the natural growth of population. The latter was negative in most regions of Russia and largely affected the main ethnic groups.

Thank You for Your attention!