Much Algebra, Little Economics

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Introduction

This paper has been stimulated by

- a recent World Bank Working Paper issued on March 2013.

«*Services Linkages and the Value Added Content of Trade*» by J. Francois, M. Manchin and P. Tomberger.

Policy Research Working Paper 6432

Introduction

- ……. as well as by a number of papers presented at the IIOA Conference held in Bratislava last year.
- …… and above all by the emphasis posed on ‘linkages’ involving value added.

Value added in Input-output

- Value added is located in the ‘supply side’ of the input-output table
- Behind the value added, primary inputs are seen through their compensation
- Labor, capital and government are not ‘measured’ in the value added
- Only data on labour compensation, interest, taxes, operating surplus and so on are reported.

An historical note

The Scandinavian model

The Scandinavian model

- After the second world war, the performance of the Swedish economy was sluggish and suffering a trade balance in disarray

- At that time, a gradual shift in emphasis on aggregate demand policy to emphasis on **supply or cost factors** took place.

The Scandinavian model

Representative of:

- the withe-collar confederation union (TCO)
- the Swedish employer association (SAF)
- the Swedish confederation of trade union (LO),

respectively Edgreen G., Faxén K. and Odhner C. built the so called Scandinavian Model known with the acronym EFO-model

The Scandinavian model

- This model begins with the division of the economy in two sectors (which can be considered as a first step towards a multisectoral representation of the economy): an open sector and a sheltered sector with respect to the international trade.
- The first sector is price-taker; the second sector is not.

The Scandinavian model

- Due to the interdependence between the two sectors, the inflation generated by the sheltered sector may hit the price competitiveness of the open sector
- The rule established was that the wage growth per unit of output in the open sector and the sheltered sector should have been the same.

The Scandinavian model

- the EFO-model became the backbone of ‘incomes policy’ applied in many countries in the years to come.
- EFO-model appears to be a ‘pure cost-push’ model which assumes accommodating demand adjustment; it has a structure of a <u>two industry economy</u>
- only the ‘supply side’ of the economy is modeled.

Demand side and Supply side

- The Input-output table collects data of the demand side as well as the sypply side of an economy
- The input-output table may be submitted to simple algebraic transformations in the field of linear algebra
- **Linkages** are defined inspecting and looking for ‘economic interpretation’ of such algebraic transformations

Basic Notation

$\mathbf{Z}$ matrix of the Intermediate Consumption

$\mathbf{f}$ vector of Final Demand

$\mathbf{v}$ vector of Value Added

$\mathbf{V}$ matrix of Value Added

$\mathbf{x}$ vector of Total Output

$\boldsymbol{\iota}$ sum vector

Basic Identities ……..

Row sum

$$x = Z^{*}\iota + f$$

Column sum

$$x' = Z'^{*}\iota + v'$$

……..and Basic Manipulations

technical coefficients

$$A = Z^{*}\hat{x}^{-1}$$

allocation coefficients

$$C = \hat{x}^{-1}{*}Z$$

GHOSH

$$CC = \hat{x}^{-1}{*}(Z\ f)$$

- Ghosh points out that an “advanced capitalistic economy’ and a ‘planned economy under centralized control’ may be modeled on the base of the same data set: an input-output table

- Are linkages neutral with respect the modeling approach?

- Technical coefficients and allocation coefficients support different ‘economic flavour’ linkages like the Leontief inverse and the Output inverse

Linkages, Identity models and Identity-centered models

Identity model

The identity

$$x = Ax + f$$

The model – n equations – n variables on the LHS and n variables on the RHS

$$x - Ax = f \quad \text{or} \quad (I - A)x = f$$

Identity model solution

Assuming x endogenous and f exogenous

The solution is

$$x = (I - A)^{-1} f$$

'**Linkages**' between x and f are investigated inspecting matrices

$$A, (I - A) \text{ and } (I - A)^{-1}$$

Identity-centered model

Identity-centered Modeling in the Accountant of SNA Based Models

Clopper Almon
Third World Inforum Conference
Lodz, Poland, 1995

Identity-centered model

the Miyazawa's example

The identity

$$x = Ax + f_c + f_o$$

Now x and f_c are endogenous

The identity

$$(I - A)x = f_c + f_o$$

Identity-centered model
the Miyazawa's example

- Assuming

$$f_c = CVx$$

- where
- C is a matrix of propensity to consume parameters
- V is the value added matrix with primary inputs income distributed in income groups for each industry

Identity-centered model

the Miyazawa's example

The identity becomes

$$x = Ax + CVx + f_o$$

And the 'solution' is

$$x = (I - A - CV)^{-1} f_o$$

Miyzawa proposes a manipulation of that inverse so that the solution may be written as follows

$$x = (I - A)^{-1}(I - CV(I - A)^{-1})^{-1} f_o$$

Identity-centered model

the Miyazawa's example

Comment:

Miyazawa argues that in his «Generalization of the Input-output Model» it is still possible to have the Leontief inverse together with a matrix named *subjoined inverse matrix*

Notwithstanding the Miyzawa's analitycal insights to give evidence of the Leontief inverse, linkages are no longer confined to matrix A.

Identity-centered model

- Miyzawa's approach (1973) is mostly based on input-output sub-matrices

- Approaches– Almon's approach (1966) was mostly supported by econometric estimated relationships

Identity-centered model
the Almon's approach

- 7 years before Miyzawa's linkages between value added and final demand components (household consumption), Almon published the real side of a multisectoral model with clear hints evoking 'supply side' variables.

Identity-centered model
the Almon's approach

- Almon's (1966) announces that his model is:

- " …. a new system for making long-range forecasts ……[and] ……. it incorporates many detailed forecasts which provide background for a variety of business decision."

Identity-centered model
the Almon's approach

…. and he declares:

"Whither the consumers' dollar leads, the American economy follows; and where our consumption projections go, the rest of the model trails along."

The end of an era

- 1973 oil crisis

- The revival of the supply-side

Identity-centered model

the Inforum type model

Confining the attention to the real side of the model and, for sake of simplicity, assuming all the equations of the real side of the model transformed into their linear approximation, the set of the final demand component equations may be exemplified as follows

$$F = Cx + DF + W_1 x_{-1} + W_2 F_{-1} + P z_R$$

Identity-centered model

the Inforum type model

Besides the final demand set of equations

$$F = Cx + DF + W_1 x_{-1} + W_2 F_{-1} + Pz_R$$

there is the set of identity equations

$$x = Ax + BF + f_{exog}$$

where f_{exog} are final demand component not modeled (exogenous); for example, government expenditure

B is a matrix of bridge matrices

Identity-centered model

the Inforum type model

In matrix notation, the Inforum type model may be seen as follows

$$\begin{bmatrix} X \\ F \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ W_1 & W_2 \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix}_{-1} + \begin{bmatrix} f_{exog} \\ Pz_R \end{bmatrix}$$

And its solution is

$$\begin{bmatrix} X \\ F \end{bmatrix} = \begin{bmatrix} (I - A) & -B \\ -C & (I - D) \end{bmatrix}^{-1} \left[\begin{bmatrix} 0 & 0 \\ W_1 & W_2 \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix}_{-1} + \begin{bmatrix} f_{exog} \\ Pz_R \end{bmatrix} \right]$$

Identity-centered model

the Inforum type model

Given the inverse of the partitioned matrix

$$\begin{bmatrix} (\mathrm{I}-\mathrm{A}) & -\mathrm{B} \\ -\mathrm{C} & (\mathrm{I}-\mathrm{D}) \end{bmatrix}^{-1} = \begin{bmatrix} \mathrm{H}_{11} & \mathrm{H}_{12} \\ \mathrm{H}_{21} & \mathrm{H}_{22} \end{bmatrix}$$

From which the «impact» of the residual exogenous final demand is

$$x = H_{11} f_{exog}$$

with

$$H_{11} = [(I - A) - B(I-D)^{-1} C]^{-1}$$

Linkages and multiplier

Matrix H_{11} may be defined as the solution of the Leontief equation residual with respect to exogenous final demand components, f_{exog}

Like the Miyazawa's solution matrix for exogenous final demand vectors, f_o, matrix H_{11} is a *solution* not an *emulsion*.

Solution and Emulsion

A **Solution**

to prepare spaghetti

Water and Salt → Salted water

An **Emulsion**

Olive oil and Vinegar → Italian salad dressing

Linkages for value added

From the accounting point of view, GDP is equal to the total final demand as well as to the total of value added.

$$\mathrm{GDP} = \iota' \mathrm{V} = \iota' \mathrm{f}$$

Linkages for value added

The above identity between the production of commodities and services and the related generated income may suggest insights about the value added content of total output different uses.

Linkages with value added

At present, the analytical approach to such linkages appears basically the same as that used in a recent World Bank working paper by J.Francois, M. Manchin, P. Tomberger (2013).

Measurement of such linkages

The measurement of the linkage starts from the Leontief equation and its solution $x=(I-A)^{-1}f$.

The connection of these flows that belong to the real side of any input-output based model is established by means of a vector of value added shares, $v_j = V_j/x_j$, where V_j is the total value added for sector j.

Given V, vector of value added by industry, and v, vector of value added shares by industry, the relationship

$$V = \hat{v}\, x$$

(where $\hat{v}$ is a diagonal matrix with the value added shares along the main diagonal) is the benchmark relationship which links value added to any final demand component

In Oosterhaven, Rueda-Cantuche (2012), the value added by industry embodied in the final demand components is defined as follows;

$$\mathrm{V} = \hat{\mathrm{v}}\,\mathrm{x} = \hat{\mathrm{v}}(\mathrm{I} - \mathrm{A})^{-1}\,\mathrm{f} = \hat{\mathrm{v}}(\mathrm{I} - \mathrm{A})^{-1}(\mathrm{f}_1 + \mathrm{f}_2 + \cdots \mathrm{f}_\mathrm{k} + \cdots + \mathrm{f}_\mathrm{K})$$

with

$$\mathrm{V}_\mathrm{k} = \hat{\mathrm{v}}(\mathrm{I} - \mathrm{A})^{-1}\mathrm{f}_\mathrm{k}$$

In Cappariello (2012) the value added embodied in the final demand components is defined as follows:

$$V_1 = \hat{v}\, f_1 \qquad V_2 = \hat{v}\, f_2 \ldots$$

$$V_k = \hat{v}\, f_k \quad \ldots . V_K = \hat{v}\, f_K$$

where the value added is not totally distributed (linked) to the final demand.

Understanding the economic structure

Hence, the measurement of such linkages is expected to give rise to many indexes as it is the case of the linkages based on the matrix of technical coefficients, A (Miller, Blair, 2009, pag. 555-582) .

Understanding the economic structure

At present, from the literature, any linkage involving the value added goes through the Leontief equation; namely, it is filtered through the real side of any interindustry model. That is to say, travelling across variables measured in real term we approach to variables strictly nominal (those modeled in the nominal side of an interindustry model).

Understanding the structural change?

If linkages between variable necessarily measured in constant prices (that is to say, the variables in the Leontief equation) and those nested in the value added are measured and compared in a time series of input-output tables, the impact of prices cannot be ignored.

Valuation over time

A theorist suggesting such comparisons may plea the *caveat* : “Valuation is at a given set of prices”.

Economists investigating structural changes cannot ignore the impact of prices in designing linkages between real side and nominal side variables.

- Miyazawa attenti alle matric C eV over time

About Linkages

- "Understanding and Interpreting Economic Structure" is a descriptive title of a collection of papers edited by G. D. Hewings, M. Sonis, M. Madden and Y. Kimura (1999). These papers offer many example of what can be obtained by applying 'linkages' indexes to input-ouput tables.

Conclusions

- Input-output Analysis is the proper field for linkages.

- As soon as an economist enters the field of Input-output modeling, the relevance of insights pursued by means of linkages fade away

Final comment

….for a model builder

Much algebra,

Little economics

Thank you for your attention