# CONSUMPTION/SAVINGS, INVESTMENT AND THE LABOUR MARKET IN THE CEPM AUSTRALIAN NATIONAL IO-ECONOMETRIC MODEL: ECONOMETRIC RESULTS

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## INTRODUCTION

This paper documents some econometric results obtained for certain inter-related macro and sectoral components of the national model. The focus of this paper will be in highlighting the econometric results obtained for household final consumption/saving behaviour, investment behaviour, potential output calculation and the labour market module of the model.

## (A) HOUSEHOLD CONSUMPTION/SAVING

### HOUSEHOLD CONSUMPTION

Household consumption (household final consumption expenditure by consumption category) is modelled using a non-linear simultaneous estimation algorithm. The specific consumption categories comprising the consumer demand system are:

1 Food
2 Cigarettes and Tobacco
3 Alcoholic Beverages
4 Clothing and Footwear
5 Rent and Other Dwelling Services
6 Electricity, Gas and Other Fuels
7 Furnishings and Household Equipment
8 Health
9 Purchase of Vehicles (G) (g)
10 Operation of Vehicles (G) (g)
11 Transport Services (G)
12 Communications
13 Recreation and Culture
14 Education Services
15 Hotels, Cafes and Restaurants
16 Insurance and Other Financial Services
17 Other Goods and Services

### Estimation of household final consumption spending by consumption category

The Perhaps Adequate Demand System (PADS) outlined in Almon (1996) is utilised to model household final consumption expenditure. This framework was adopted because of its ability to deal with the impact on household final consumption expenditure of growth in real income and changes in demographic effects and relative prices that permit both complementarity and substitution between different goods and services. The functional form adopted for estimation is given by:

$$pce_{i,t} = \left(\alpha_0 + \alpha_1 * hhyd_t + \alpha_2 * \Delta hhyd_t + \alpha_3 * time\right)\left(\frac{p_i}{P}\right)^{-\lambda_i} \prod_{k=1}^{n}\left(\frac{p_i}{p_k}\right)^{-\lambda_k s_k}\left(\frac{p_i}{p_G}\right)^{-\mu_G}\left(\frac{p_i}{p_g}\right)^{-\nu_g}$$

where:

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. $pce_{i,t}$ is real consumption of commodity i per capita in period t;
. $hhyd_t$ is real disposable income per capita;
. $\Delta$ is the first difference operator;
. $P$ is the overall consumption price index;
. $p_i$ is the price index of commodity i;
. $p_G$ is average price index of group G;
. $p_g$ is average price index of sub-group g;
. $\lambda_i$ is price response parameter for commodity i;
. $\mu_G$ is price response parameter for group G;
. $v_g$ is price response parameter for sub-group g; and
. $s_i$ is budget share of commodity i in the base year.

This functional form introduces a multiplicative relationship between the income and price terms. Slutsky symmetry is imposed in the base year and will continue to hold in an approximate sense as long as the shares of the various products in total expenditure do not change significantly from the shares in the base year. Finally, in order to free up effective degrees of freedom (to reduce the number of cross price coefficients that need to be estimated) symmetry is imposed on the cross price coefficients, and commodity groupings and sub-groupings are also employed. The groupings and sub-groupings are used to capture close linkages between commodities that are likely to exhibit strong complementarity or substitutability effects but have little inter-relationship outside the group. The total number of price related coefficients that need to be estimated is equal to the number of commodities (each has its own price elasticity) plus the number of groups plus the number of sub-groups.

It should be noted that in the results reported below, we define commodities in the “transport” sector as a commodity group (combining consumption categories 9 to 11), and, within this group, define the sector “vehicles” as a sub-group (combining commodities 9 and 10). These details are defined in the Groups.ttl file whose contents are displayed in Table 1, (see columns 2 and 3). The population concept employed is total estimated resident population.

It is possible that some commodities will display very little price sensitivity and estimation based on relative prices will not adequately fit the data. In this case, commodities are handled outside of the system. For these “price insensitive” commodities, all own and cross price relationships are set to zero and price terms are essentially dropped from the estimated equations. Moreover, they are constrained to have no impact on the “price sensitive” commodities. In the results reported below, no commodity categories are handled outside the system – see column 7 of Table 1.

The estimation algorithm used is a simultaneous non-linear estimation technique based on the Marquardt algorithm with adjustment that permits the incorporation of soft constraints in the estimation process (see Almon 1996, pp.10-13). These soft constraints are typically used to impose constraints on the income and own price elasticities to ensure that they are consistent with theoretical considerations – ie. to ensure that income elasticities are positive and own price elasticities are negative. Soft constraints can also be used to impose substitution or complementarity relationships between commodities comprising groups or sub-groups. Soft constraints are also employed to ensure that the magnitude of any negative coefficient on $\Delta hhyd_t$ is smaller (in absolute value) than the magnitude of the (positive) estimated income elasticity (on $hhyd_t$), thus ensuring that real per capita consumption increases with growth in real income.

Note that the soft constraints employed in estimation to determine the results reported below are documented in Table 2. For each regressor variable, i.e. Income, DIncome etc, two numbers are displayed. The first is the value of the constraint while the second is the weighting given to the constraint relative to the value determined from the data. In the latter case, a value of one implies that equal weight is given to the value implied by the constraint and the value determined from the data. A value less than one implies more relative weight is given to what the data determines relative to the coefficient value implied in the constraint itself.

Table 2 shows that only one constraint is applied to the income elasticities, for sector 10 “Operation of Vehicles” (see the “Income” column in Table 2). Two constraints are applied for the change in income term, for sectors 8 and 10 corresponding to “Health” and “Operation of Vehicles” (see the DIncome column in Table 2). In these three cases, a weight of one is adopted giving equal weighting to the constraint value and coefficient value determined from the estimation process. No constraints are applied to the time trend terms. Seven constraints in all are imposed to ensure the own price elasticities are negative and/or of reasonable magnitude – see the “lambda” column in Table 2. For these latter constraints, all except one (corresponding to sector 10) have a weighting less than one signifying that relatively more weight is attached to what the data determines relative to the value of the constraint itself. Overall, both the number of constraints and the weights attached to the constraints signify that the system is not heavily constrained.

With the soft constraints documented in Table 2, acceptable results are achieved. These results are documented in Table 3. There is only one sector, Health Services (8), which shows a standard error above 7% and only four sectors (8, 9, 12 and 16) have standard errors above 5%. All income and price elasticities have correct signs and plausible magnitudes. The time trends have magnitudes less than 3% (in absolute value) for all goods except Cigarettes and Tobacco (2). Moreover, the reasonably strong negative trends for sectors 2 and 3 are not unreasonable given health related advertising about the possible adverse health consequences of consumption of tobacco and alcoholic beverages and they do not mitigate the reasonably strong income elasticities observed for these particular commodity categories. Finally, note that the absolute values of all negative coefficients on the change in disposable income (the Dinc term in Table 3) are smaller than the corresponding magnitudes of the income elasticity (IncEl) terms. Recall that in order to achieve this, sectors 8 and 10 had to be softly constrained – see Table 2.

The cross price elasticities for the “transport” group and “vehicle” sub-group are reported in Table 4. The negative signs of the elasticities for the vehicle sub-group (commodities 9 and 10) signify strong complementarity between these commodities (purchase of vehicles and operation of vehicles). However, both of these commodities are substitutes with commodity 11, transport services, which principally relates to consumption of public transport services.

**Table 1. The groups file for PADS estimation**

```
# Groups.ttl.  Columns are
# 1  The consumption category number
#    2  The group number
#       3  The subgroup number
#          4  The weighted population number to be used with this category
#             5  The income
#                6 The time trend
#                   7 Include in system (sensitive = 1)
#                     8 The title of the category
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 4 | 1 | 1 | 1 | Food |
| 2 | 0 | 0 | 4 | 1 | 1 | 1 | Cigarettes and tobacco |
| 3 | 0 | 0 | 4 | 1 | 1 | 1 | Alcoholic beverages |
| 4 | 0 | 0 | 4 | 1 | 1 | 1 | Clothing and footware |
| 5 | 0 | 0 | 4 | 1 | 1 | 1 | rent and other dwelling services |
| 6 | 0 | 0 | 4 | 1 | 1 | 1 | Electricity, gas and other fuels |
| 7 | 0 | 0 | 4 | 1 | 1 | 1 | Furnishings, household equip, etc |
| 8 | 0 | 0 | 4 | 1 | 1 | 1 | Health |
| 9 | 1 | 1 | 4 | 1 | 1 | 1 | Purchase of vehicles |
| 10 | 1 | 1 | 4 | 1 | 1 | 1 | Operation of vehicles |
| 11 | 1 | 0 | 4 | 1 | 1 | 1 | Transport services |
| 12 | 0 | 0 | 4 | 1 | 1 | 1 | Communications |
| 13 | 0 | 0 | 4 | 1 | 1 | 1 | Recreation and culture |
| 14 | 0 | 0 | 4 | 1 | 1 | 1 | Education services |
| 15 | 0 | 0 | 4 | 1 | 1 | 1 | Hotels, cafes and restaurants |
| 16 | 0 | 0 | 4 | 1 | 1 | 1 | Insurance and other financial services |
| 17 | 0 | 0 | 4 | 1 | 1 | 1 | Other goods and services |

**Table 2. The softcon file for PADS estimation**

| #sec | Title | Income | | DIncome | | Time | | lambda | | mu | | nu | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Food | .05 | 0 | -.05 | 0 | .01 | 0 | .1 | .5 | | | | |
| 2 | Cigarettes and tobacc | .05 | 0 | -.05 | 0 | 0 | 0 | .1 | 0 | | | | |
| 3 | Alcoholic beverages | .05 | 0 | -.05 | 0 | 0 | 0 | .1 | .5 | | | | |
| 4 | Clothing and footwear | .05 | 0 | 0 | 0 | -.01 | 0 | .1 | 0 | | | | |
| 5 | Rent and oth dwell serv | 0 | 0 | -.05 | 0 | 1 | 0 | .1 | .5 | | | | |
| 6 | Elect, gas and oth fuel | 0 | 0 | -.05 | 0 | .01 | 0 | 0 | 0 | | | | |
| 7 | Furnish, hh equip, etc | .05 | 0 | -.05 | 0 | .01 | 0 | .1 | 0 | | | | |
| 8 | Health | .05 | 0 | -.2 | 1 | .01 | 0 | .1 | 0 | | | | |
| 9 | Purchases of vehicles | .05 | 0 | .9 | 0 | 0 | 0 | .1 | 0 | .1 | 0 | -.5 | 0 |
| 10 | Operation of vehicles | .5 | 1 | -.1 | 1 | .01 | 0 | .3 | 1 | | | | |
| 11 | Transport services | .05 | 0 | 0 | 0 | -.01 | 0 | .1 | 0 | | | | |
| 12 | Communications | .05 | 0 | 0 | 0 | 0 | 0 | .1 | 0 | | | | |
| 13 | Recreation and culture | 0 | 0 | 0 | 0 | .01 | 0 | 0 | 0 | | | | |
| 14 | Education services | 0 | 0 | -.01 | 0 | 2 | 0 | 0 | 0 | | | | |
| 15 | Hotels, cafes and restau | 1 | 0 | .01 | 0 | -.01 | 0 | .1 | .5 | | | | |
| 16 | Insurance, oth fin serv | .1 | 0 | -.05 | 0 | -.01 | 0 | .1 | .5 | | | | |
| 17 | Other goods and services | .1 | 0 | -.05 | 0 | 1 | 0 | .1 | .5 | | | | |

**Table 3. Results of PADS by consumption category**

The value of L is 0.27
mu: 0.58
nu: -0.96

| nsec | title | G | S | P | C | T | I | lamb | share | IncEl | DInc | time% | PrEl | Err% | rho |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Food | 0 | 0 | 4 | 1 | 1 | 1 | 0.06 | 0.111 | 0.44 | -0.13 | 0.20 | -0.31 | 1.92 | 0.77 |
| 2 | Cigarettes and tobacco | 0 | 0 | 4 | 1 | 1 | 1 | 0.11 | 0.020 | 1.55 | -0.33 | -3.17 | -0.37 | 3.14 | 0.43 |
| 3 | Alcoholic beverages | 0 | 0 | 4 | 1 | 1 | 1 | 0.06 | 0.019 | 2.26 | -0.36 | -2.78 | -0.33 | 3.14 | 0.62 |
| 4 | Clothing and footware | 0 | 0 | 4 | 1 | 1 | 1 | 0.19 | 0.040 | 1.08 | -0.31 | -1.06 | -0.44 | 3.68 | 0.61 |
| 5 | rent and other dwelling | 0 | 0 | 4 | 1 | 1 | 1 | 0.08 | 0.189 | 0.30 | -0.16 | 1.88 | -0.32 | 0.89 | 0.81 |
| 6 | Electricity, gas and ot | 0 | 0 | 4 | 1 | 1 | 1 | -0.09 | 0.020 | 1.20 | -0.27 | 0.03 | -0.18 | 2.30 | 0.63 |
| 7 | Furnishings, household | 0 | 0 | 4 | 1 | 1 | 1 | 0.01 | 0.058 | 1.13 | 0.03 | -0.59 | -0.28 | 2.90 | 0.56 |
| 8 | Health | 0 | 0 | 4 | 1 | 1 | 1 | -0.07 | 0.042 | 0.60 | -0.32 | 0.37 | -0.20 | 7.07 | 0.57 |
| 9 | Purchase of vehicles | 1 | 1 | 4 | 1 | 1 | 1 | 0.37 | 0.040 | 3.01 | -0.06 | -1.47 | -0.42 | 6.21 | 0.46 |
| 10 | Operation of vehicles | 1 | 1 | 4 | 1 | 1 | 1 | 0.22 | 0.060 | 0.45 | -0.24 | 0.85 | -0.37 | 2.99 | 0.82 |
| 11 | Transport services | 1 | 0 | 4 | 1 | 1 | 1 | 0.24 | 0.024 | 3.30 | -0.53 | -1.54 | -0.96 | 3.70 | 0.60 |
| 12 | Communications | 0 | 0 | 4 | 1 | 1 | 1 | 0.99 | 0.023 | 2.96 | -0.04 | 0.25 | -1.22 | 5.56 | 0.71 |
| 13 | Recreation and culture | 0 | 0 | 4 | 1 | 1 | 1 | 1.46 | 0.123 | 1.02 | 0.13 | 0.72 | -1.38 | 2.93 | 0.79 |
| 14 | Education services | 0 | 0 | 4 | 1 | 1 | 1 | 0.24 | 0.022 | 0.95 | -0.74 | 2.30 | -0.50 | 2.48 | 0.52 |
| 15 | Hotels, cafes and resta | 0 | 0 | 4 | 1 | 1 | 1 | 0.05 | 0.070 | 2.06 | -0.10 | -1.05 | -0.31 | 2.60 | 0.30 |
| 16 | Insurance and other fin | 0 | 0 | 4 | 1 | 1 | 1 | -0.15 | 0.064 | 2.07 | -0.78 | 0.26 | -0.13 | 6.79 | 0.61 |
| 17 | Other goods and service | 0 | 0 | 4 | 1 | 1 | 1 | 0.09 | 0.075 | 0.75 | -0.09 | 1.74 | -0.35 | 3.16 | 0.79 |

**Table 4. Cross price elasticities of "transport" Group and "vehicles" sub-group[2]**

| | 9 | 10 | 11 |
|---|---|---|---|
| 9 | -0.42 | -0.24 | 0.13 |
| 10 | -0.15 | -0.37 | 0.13 |
| 11 | 0.22 | 0.32 | -0.96 |

[2] The number in row i and column j is the elasticity of product i with respect to the price of product j.

After household final consumption expenditure by consumption category has been determined in the model, this vector is passed through a consumption bridge to obtain the disposition (sales) of each IO industry's output that accrues to household final consumption expenditure.

**HOUSEHOLD SAVINGS**[3]

**Key Endogenous variables:**

. Gross Household savings level and rate
. Net Household savings level and rate
. Household consumption of fixed capital

**Discussion**

There are currently two ways that gross and net saving can be determined in the model. The first way envisages savings as being determined econometrically, and then subtracted from household gross disposable income to derive a control total for total household final consumption spending, which then feeds into the PADS system to determine household final consumption spending by consumption category.

Within this broad framework, two particular scenarios are adopted relating to the modelling of gross and net savings. First, rates of gross and net saving are estimated econometrically, converted to levels and household consumption of fixed capital is calculated as a residual item – ie. as the difference between gross and net savings. Two functional forms have been used to model the savings rate relationships:

$$savrat_t = \beta_0 + \underset{(-)}{\beta_1 * unemp_{t-1}} + \underset{(+)}{\beta_2 * rcb_{t-1}} + \underset{(+)}{\beta_3 * hhpcti_t} + \underset{(+)}{\beta_4 * grpcon_t} + \underset{(-)}{\beta_5 * sdum_t},$$

and

$$savrat_t = \beta_0 + \underset{(+)}{\beta_1 * (gdpgap_t - 100)} + \underset{(+)}{\beta_2 * rcb_{t-1}} + \underset{(-)}{\beta_3 * sdum1_t},$$

where:

. $savrat_t$ is the savings rate;
. $unemp_t$ is the unemployment rate;
. $rcb_t$ is the short term interest rate (90-day accepted commercial bill rate);
. $hhpcti_t$ is growth in real household gross disposable income;
. $grpcon_t$ is growth in the household final consumption expenditure price deflator;
. $(gdpgap_t - 100)$ is an index which takes a value greater than zero when actual GDP exceeds potential GDP (i.e. the economy is "tight"); and
. $sdum_t$ and $sdum1_t$ are dummy variables that account for the observed decline in the household saving rate since the mid 1970's. Variable $sdum_t$ takes a value of 1 after 1974 while variable $sdum1_t$ takes a value of 1 after 1978.

The signs of the coefficients indicate that we expect the savings rate to be negatively related to the rate of unemployment and positively related to the

[3] See McCarthy (1991) and Meade (2000, pp.14-15).

gdpgap index variable. These variables are designed to pick up the impact that overall economic growth will have on the level of household saving, which is expected to increase relative to household disposable income during periods of economic prosperity. We also expect the household saving rate to be positively related to short-term interest rates because of the increase in interest income that can be derived from deposits when interest rates are rising. Finally, increases in both real household disposable income and consumer inflation will contribute to increases in nominal household income and the inflation premium in nominal interest rates, thereby producing an accompanying increase in the level of savings.

The coefficient estimates for the **first** functional form outlined above are:

**Gross savings rate equation**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | 14.770 |
| $\beta_1$ | -0.329 |
| $\beta_2$ | 0.176 |
| $\beta_3$ | 0.398 |
| $\beta_4$ | 0.718 |
| $\beta_5$ | -3.290 |

$R^2$ = 0.79, MAPE = 9.35, SEE = 1.73, RHO = 0.60.

**Net savings rate equation**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | 4.745 |
| $\beta_1$ | -0.247 |
| $\beta_2$ | 0.179 |
| $\beta_3$ | 0.240 |
| $\beta_4$ | 0.736 |
| $\beta_5$ | -1.916 |

$R^2$ = 0.73, MAPE = 27.66, SEE = 1.87, RHO = 0.63.

The coefficient estimates for the **second** functional form outlined above are:

**Gross saving rate equation**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | 12.672 |
| $\beta_1$ | 0.735 |
| $\beta_2$ | 0.706 |
| $\beta_3$ | -4.155 |

$R^2$ = 0.44, MAPE = 15.23, SEE = 2.70, RHO = 0.72.

**Net saving rate equation**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | 4.648 |
| $\beta_1$ | 0.734 |
| $\beta_2$ | 0.700 |
| $\beta_3$ | -4.234 |

$R^2$ = 0.38, MAPE = 48.91, SEE = 2.95, RHO = 0.77.

The t statistics are not listed above because soft constraints were employed in the estimation of the above equations. This observation follows because the use of soft constraints invalidates the conventional interpretation of t statistics. All equations were estimated by OLS using the “r” command in the g7 econometric package.

The second approach to modelling gross and net savings within the first broad framework mentioned above is to estimate the gross saving rate econometrically and convert to levels together with household consumption of fixed capital, and then calculate the level of net savings as a residual item – in this case, as the difference between the level of gross savings and household consumption of fixed capital. The specifications for the gross savings rate are as above. The functional form adopted for household consumption of fixed capital is:

$$hhcfc_t = \beta_0 + \beta_1 * gdpr_t + \beta_2 * gdpr_{t-1} + \beta_3 * gdpr_{t-2} + \beta_4 * gdp\_ipd_t + \beta_5 * gdp\_ipd_{t-1} + \beta_6 * gdp\_ipd_{t-2} + \beta_7 * time$$

where the sum of the sets of coefficients $\beta_1, \beta_2, \beta_3$ and $\beta_4, \beta_5, \beta_6$ would be expected to be positive although constraints were not necessary in estimation to achieve this. Real GDP $(gdpr_t)$ and the GDP price deflator $(gdp\_ipd)$ were used to separate out the effects on the dependent variable of volume and price effects that jointly combine to determine nominal GDP. The rationale for using GDP is as an overall measure of economic activity. We expect household consumption of fixed capital to be positively linked to growth in the economy.

The coefficient estimates for the household consumption of fixed capital equation are:

**Household consumption of fixed capital equation**

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\beta_0$ | -29281.305 | -14.62 |
| $\beta_1$ | 0.107 | 5.26 |
| $\beta_2$ | -0.024 | -0.83 |
| $\beta_3$ | 0.044 | 1.99 |
| $\beta_4$ | 87810.115 | 6.21 |
| $\beta_5$ | -104606.426 | -3.98 |
| $\beta_6$ | 66755.392 | 4.35 |
| $\beta_5$ | -2043.262 | -8.55 |

$R^2$ = 0.996, MAPE = 3.19, SEE = 564.18, RHO = 0.11.

The second way of modelling savings in the model is essentially to treat savings as a residual item. In this case, given household gross disposable income and total household final consumption spending obtained after applying the PADS system, gross savings is calculated as a residual item. Net savings is obtained after estimating household consumption of fixed capital and then subtracting this from the level of gross savings. The functional form adopted for the household consumption of fixed capital equation is as mentioned above.

## (B) GROSS FIXED CAPITAL EXPENDITURE (GFCF) (BY PURCHASING INDUSTRY)

### NON-DWELLING INVESTMENT EXPENDITURE[4]

Non-dwelling gross fixed capital expenditure collectively incorporates:

- . Investment in plant and equipment;
- . Investment in non-residential structures and
- . Investments in:
  - - artistic originals;
  - - computer software;
  - - mineral exploration; and
  - - livestock.

The purchasing industries associated with non-dwelling investment are:

1 Agriculture, Forestry and Fishing
2 Coal, Oil and Gas Mining
3 Other Mining
4 Food, Beverage and Tobacco Manufacturing
5 Textiles, Clothing and Footwear Manufacturing
6 Wood, Paper and Print Manufacturing
7 Petroleum, Chemicals and Associated Product Manufacturing
8 Non-Metallic Product Manufacturing
9 Metal Product Manufacturing
10 Machinery and Equipment Manufacturing
11 Other Manufacturing
12 Electricity, Gas and Water
13 Construction
14 Trade
15 Accommodation, Cafes and Restaurants
16 Transport and Storage
17 Communications
18 Finance and Insurance
19 Business and Other Property Services
20 Government Administration and Defense
21 Education Services
22 Health and Community Services
23 Cultural and Recreational Services
24 Personal and Other Services

Many investment models require an estimate of the productive capital stock for the purchasing industry. This capital stock concept is determined using a second order pascal lag distribution. This distribution can be interpreted in terms of the following two-bucket scheme:

$$B_1(t) = I_t + \lambda * B_1(t-1)$$
$$B_2(t) = (1-\lambda) * B_1(t) + \lambda * B_2(t-1)$$

where $I_t$ is gross fixed capital expenditure for the purchasing industry, and $\lambda$ is the depreciation rate which is set equal to 2/average age of the gross capital stock for this sector. The productive capital stock is calculated as the sum of the two buckets:

[4] The discussion and motivation for the material in this section was obtained from reading Douglas Meade's 1990 Ph. D dissertion, titled "Investment in a Macroeconometric Interindustry Model". See Meade (1990).

$$K(t) = B_1(t) + B_2(t)\ ,$$

consult Meade (1990, pp. 437-441).

Many models also require the user cost of capital. This cost concept is imputed using the opportunity cost of capital – the rate of return, depreciation and price index for capital goods. Elements of the tax system such as the corporate tax rate, investment tax credits and tax treatment of depreciation are also taken into account because they can affect the rate of return received by investors. The following formula is used to calculate the user cost of capital:

$$c = p(r+\delta) * \frac{(1-TZ-C)}{(1-T)}\ ,$$

where:

- . $c$ is the user cost of capital;
- . $p$ is the price index of capital goods (the gross fixed capital expenditure price deflator of the purchasing industry);
- . $r$ is the real 10 year treasury bond rate (calculated as the nominal rate minus the expected rate of inflation) which denotes the opportunity cost of capital;
- . $\delta$ is the rate of depreciation (set equal to the spill rate used when calculating the productive capital stock);
- . $T$ is the corporate tax rate;
- . $Z$ is the discounted present value of one dollar's worth of depreciation deductions (assuming straight-line depreciation); and
- . $C$ is an investment tax credit.

See Meade (1990, pp. 442-446) and Meade (1996, pp.4-5).

**Models Employed in Estimation**

**(1) Generalised Leontief Model (for joint investment and labour demand by purchasing industry)**

The Generalised Leontief (GL) Putty-Clay model is a two-stage, two-equation framework in which factor demands for capital and labour are estimated simultaneously (see Meade 1990, pp. 126-132). The first stage equations for optimal capital-output and labour-output ratios are obtained using Shephard's Lemma with a generalized Leontief cost function with capital and labour inputs. The first stage optimal capital-output and labour-output ratios are estimated from:

$$\left(\frac{K}{Q}\right)_t^* = e^{-a_K t}\left\{\sum_{j=K,L} b_{Kj}\left(\frac{p_j}{p_K}\right)^{\frac{1}{2}}\right\}$$

$$\left(\frac{L}{Q}\right)_t^* = e^{-a_L t}\left\{\sum_{j=K,L} b_{Lj}\left(\frac{p_j}{p_L}\right)^{\frac{1}{2}}\right\}$$

where:

- . $K_t =$ productive capital stock;
- . $L_t =$ employment (total hours worked);
- . $Q_t =$ output;
- . $\left(\frac{K}{Q}\right)_t^* =$ optimal capital-output ratio;
- . $\left(\frac{L}{Q}\right)_t^* =$ optimal labour-output ratio;
- . $p_j =$ price of factor *j*, where *j* = *K*, *L* and where $p_k$ is the user cost of capital ($c$) mentioned above and $p_l$ is hourly wage rate for the purchasing industry; and
- . $t =$ time trend.

These equations are used in a two-equation system to fit the historical capital-output and labor-output ratios.

The equations for net investment and employment demand are derived from the first derivative of the optimal capital/output and labour/output ratio equations and can be expressed as:

$$N_t = e^{-a_K t}\left\{\sum_{j=K,L} b_{Kj}\left(\frac{p_j}{P_K}\right)^{\frac{1}{2}}\right\}\sum_{j=0}^{3} w_j^K \Delta Q_{t-j}$$

$$L_t = e^{-a_L t}\left\{\sum_{j=K,L} b_{Lj}\left(\frac{p_j}{P_L}\right)^{\frac{1}{2}}\right\}\sum_{j=0}^{3} w_j^L Q_{t-j} ,$$

where:

- . $N_t =$ net investment;
- . $L_t$ = employment of labour services;
- . $Q_t =$ output; and
- . $\Delta Q_t =$ change in output.

In the second stage, the parameters from the first stage are treated as fixed, and the equation for net investment is based upon a distributed lag of past changes in output while the equation for labour services is based upon a distributed lag of levels of output.

The estimation algorithm used imposes strict inequality constraints on the two-equation system to ensure that the own price elasticities for capital and labour input are negative in the first stage of the estimation process. Hence, this constraint will ensure that an increase in the prices of capital or labour will reduce the usage of capital and labour input, respectively. In the second stage of the estimation process, strict inequality constraints are imposed to ensure that the sum of the coefficients of the distributed lags of changes in output (and output) sum to positive values that are between 0.2 and 1.0. Note that if these sums are close to unity, this will produce a one-to-one relationship between increases in the actual levels of investment and employment produced in response to increases in the desired stock of capital and desired demand for labour services. Finally, constraints are also applied to ensure that the lag patterns on the distributed lag coefficients $w_j^K$ and $w_j^L$ decay smoothly, and are nonnegative.

The strict inequality constraints are imposed using a quadratic programming-based regression algorithm. This algorithm estimates a system of equations as one quadratic objective function subject to linear inequality constraints.

It should also be noted that because only two factors are employed, namely capital and labour, the requirement of negative own price elasticities requires that the GL framework impose a substitution relationship between the two factors of production. If three or more factors of production are employed, however, then complementarity in factor usage is possible, for example capital and energy (i.e. electricity usage) might be complementary while capital and labour remain substitutes.

Replacement investment is determined by multiplying the optimal capital output ratio by the losses to capacity (measured as the level of optimal output given the current capital stock) occurring in the current year.

Equations for investment and employment were estimated jointly for the 24 purchasing industries mentioned above. Table 5 displays the estimation results in terms of price elasticities and elasticities of substitution for the investment demand equations for the financial year 2000/2001. As mentioned previously, own price elasticities (column 2) are constrained to be negative. For example, for industry 2 (coal, oil and gas mining), a value of –0.147 means that, ceteris paribus, a 1% increase in the price of capital will result in a 0.147% decrease in the optimal capital stock. Moreover, a 1% increase in wages relative to the cost of capital will result in a 0.147% increase in the optimal capital stock. As such, labour and capital are clearly substitutes as alluded to above. The positive signs on the elasticity of substitution (column 4 in Table 5 - SKL) also signify substitutability between capital and labour. Quite a lot of the elasticities are small in magnitude with only 13 and 14 out of 24 industries having price elasticities and elasticities of substitution whose absolute values are greater than 0.1 respectively.

**Table 5. Cross price elasticities and elasticities of substitution for GL investment equations**

| | **Industry** | **PK** | **PL** | **SKL** |
|---|---|---|---|---|
| 1 | Agriculture, Forestry and Fishing | -0.003 | 0.003 | 0.014 |
| 2 | Coal Mining | -0.147 | 0.147 | 0.692 |
| 3 | Other Mining | -0.149 | 0.149 | 0.788 |
| 4 | Food, Beverage and Tobacco Manufacturing | -0.299 | 0.299 | 0.507 |
| 5 | Textiles, Clothing and Footwear Manufacturing | -0.494 | 0.494 | 0.699 |
| 6 | Wood, Paper and Print Manufacturing | -0.567 | 0.567 | 0.920 |
| 7 | Petroleum, Chemicals and Associated Product Manu | -0.081 | 0.081 | 0.150 |
| 8 | Non-Metallic Product Manufacturing | -0.305 | 0.305 | 0.646 |
| 9 | Metal Product Manufacturing | -0.151 | 0.151 | 0.252 |
| 10 | Machinery and Equipment Manufacturing | -0.033 | 0.033 | 0.049 |
| 11 | Other Manufacturing | -1.462 | 1.462 | 2.015 |
| 12 | Electricity, Gas and Water | -0.725 | 0.725 | 3.948 |
| 13 | Construction | -0.034 | 0.034 | 0.047 |
| 14 | Trade | -0.039 | 0.039 | 0.050 |
| 15 | Accommodation, Cafes and Restaurants | -0.047 | 0.047 | 0.078 |
| 16 | Transport and Storage | -0.035 | 0.035 | 0.086 |
| 17 | Communications | -1.809 | 1.809 | 4.124 |
| 18 | Finance and Insurance | -0.063 | 0.063 | 0.094 |

| | | | |
|---|---|---|---|
| 19 Business and Other Property Services | -0.049 | 0.049 | 0.066 |
| 20 Government Administration and Defense | -0.562 | 0.562 | 0.762 |
| 21 Education Services | -0.035 | 0.035 | 0.047 |
| 22 Health and Community Services | -0.688 | 0.688 | 0.871 |
| 23 Cultural and Recreational Services | -0.018 | 0.018 | 0.033 |
| 24 Personal and Other Services | -0.338 | 0.338 | 0.430 |

The estimated coefficients for the net investment equations are reported in Table 6. An intercept was included in the equations for net investment to improve both the fitting ability and forecast performance of the equations.

**Table 6. GL estimation results for stage 2 investment equations**

| Industry | PK | PL | TREND | $\Delta$ OUTP | $\Delta$ OUTP(-1) | $\Delta$ OUTP(-2) | $\Delta$ OUTP(-3) | INTCP | $\sum \Delta out$ | RSQR |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 8.354 | 0.005 | 0.026 | 0.051 | 0.063 | 0.056 | 0.030 | 0.384 | 0.200 | -0.216 |
| 2 | 4.466 | 0.044 | 0.022 | 0.088 | 0.075 | 0.018 | 0.018 | 0.981 | 0.200 | 0.047 |
| 3 | 5.610 | 0.132 | 0.009 | 0.024 | 0.059 | 0.059 | 0.059 | 0.975 | 0.200 | 0.066 |
| 4 | 0.656 | 0.100 | 0.002 | 0.396 | 0.258 | 0.258 | 0.088 | 0.074 | 1.000 | 0.592 |
| 5 | 0.181 | 0.177 | 0.012 | 0.000 | 0.081 | 0.081 | 0.042 | -0.039 | 0.203 | 0.026 |
| 6 | 0.074 | 0.199 | 0.008 | 0.402 | 0.233 | 0.231 | 0.134 | 0.002 | 1.000 | 0.373 |
| 7 | 1.324 | 0.033 | -0.007 | 0.310 | 0.298 | 0.200 | 0.000 | 0.027 | 0.808 | 0.265 |
| 8 | 0.522 | 0.151 | -0.007 | 0.041 | 0.053 | 0.053 | 0.053 | 0.034 | 0.200 | -0.034 |
| 9 | 2.302 | 0.062 | 0.015 | 0.000 | 0.170 | 0.170 | 0.170 | 0.069 | 0.509 | 0.163 |
| 10 | 0.354 | 0.009 | -0.034 | 0.394 | 0.202 | 0.202 | 0.202 | 0.011 | 1.000 | 0.516 |
| 11 | -0.483 | 0.463 | 0.031 | 0.087 | 0.086 | 0.080 | 0.000 | 0.019 | 0.253 | 0.116 |
| 12 | 12.645 | 0.790 | 0.038 | 0.000 | 0.080 | 0.060 | 0.060 | 0.757 | 0.200 | -0.264 |
| 13 | 0.487 | 0.008 | -0.016 | 0.061 | 0.145 | 0.019 | 0.019 | 0.293 | 0.243 | 0.273 |
| 14 | 0.914 | 0.010 | -0.007 | 0.211 | 0.253 | 0.203 | 0.203 | 0.238 | 0.869 | 0.677 |
| 15 | 1.187 | 0.028 | -0.017 | 0.267 | 0.476 | 0.240 | 0.018 | -0.062 | 1.000 | 0.477 |
| 16 | 11.179 | 0.031 | 0.026 | 0.084 | 0.094 | 0.065 | 0.059 | 1.450 | 0.302 | 0.153 |
| 17 | -0.676 | 0.896 | 0.043 | 0.000 | 0.395 | 0.395 | 0.210 | -0.003 | 1.000 | 0.795 |
| 18 | 0.468 | 0.011 | -0.014 | 0.057 | 0.479 | 0.464 | 0.000 | 0.082 | 1.000 | 0.637 |
| 19 | 0.474 | 0.011 | -0.024 | 0.148 | 0.336 | 0.275 | 0.241 | 0.249 | 1.000 | 0.722 |
| 20 | 2.146 | 0.175 | 0.021 | 0.109 | 0.112 | 0.047 | 0.025 | 0.532 | 0.293 | 0.056 |
| 21 | 4.305 | 0.013 | 0.028 | 0.104 | 0.059 | 0.018 | 0.018 | 0.753 | 0.200 | -0.187 |
| 22 | 0.668 | 0.161 | 0.022 | 0.336 | 0.236 | 0.236 | 0.191 | -0.056 | 1.000 | 0.391 |
| 23 | 0.553 | 0.008 | -0.034 | 0.122 | 0.341 | 0.341 | 0.196 | 0.027 | 1.000 | 0.182 |
| 24 | 0.175 | 0.077 | -0.007 | 0.171 | 0.276 | 0.276 | 0.276 | 0.036 | 1.000 | 0.659 |

Note that the $R^2$'s for many industries are low or even negative (see column 11 in Table 6 titled RSQR). This outcome can be attributed to the fact that the equations are non-linear with inequality constraints – in particular, with the constraints on both the sum and pattern of distributed lag of change in output. It is apparent from inspection of this table that 7 out of 24 industries have an $R^2$ greater than or equal to 0.5. Of the remainder, 4 industries have negative $R^2$'s. The sum of the coefficients associated with the distributed lag on the change in output terms are reported in column 10 of Table 6. These take values between 0.2 and 1.0. The coefficients in column 9 of Table 6 list the values of the intercept term in the net investment equations, divided by 1000.

The estimation results in terms of price elasticities for labour demand for the financial year 2000/01 are reported in Table 7. As was the case in the investment demand equations, the own price elasticities (column 3) are constrained to be negative. For example, for industry 4 (food, beverage and tobacco manufacturing), a value of –0.208 means that, ceteris paribus, a 1% increase in the price of labour (the hourly wage rate) will result in a 0.208% decrease in the optimal (desired) level of labour input demanded. Furthermore, a 1% increase in the price of capital relative to the price of labour services will result in a 0.208% increase in the optimal level of labour input demanded. As such, labour and capital are, once again, substitutes. It is also apparent that a lot of the elasticities are small in magnitude with only 12 out of 24 industries having price elasticities whose absolute values are greater than 0.1.

**Table 7. Labour demand price elasticities**

| | Industry | PK | PL |
|---|---|---|---|
| 1 | Agriculture, Forestry and Fishing | 0.011 | -0.011 |
| 2 | Coal Mining | 0.545 | -0.545 |
| 3 | Other Mining | 0.640 | -0.640 |
| 4 | Food, Beverage and Tobacco Manufacturing | 0.208 | -0.208 |
| 5 | Textiles, Clothing and Footwear Manufacturing | 0.205 | -0.205 |
| 6 | Wood, Paper and Print Manufacturing | 0.353 | -0.353 |
| 7 | Petroleum, Chemicals and Associated Product Manu | 0.070 | -0.070 |
| 8 | Non-Metallic Product Manufacturing | 0.341 | -0.341 |
| 9 | Metal Product Manufacturing | 0.101 | -0.101 |
| 10 | Machinery and Equipment Manufacturing | 0.015 | -0.015 |
| 11 | Other Manufacturing | 0.554 | -0.554 |
| 12 | Electricity, Gas and Water | 3.222 | -3.222 |
| 13 | Construction | 0.012 | -0.012 |
| 14 | Trade | 0.011 | -0.011 |
| 15 | Accommodation, Cafes and Restaurants | 0.031 | -0.031 |
| 16 | Transport and Storage | 0.051 | -0.051 |
| 17 | Communications | 2.315 | -2.315 |
| 18 | Finance and Insurance | 0.031 | -0.031 |
| 19 | Business and Other Property Services | 0.017 | -0.017 |
| 20 | Government Administration and Defense | 0.201 | -0.201 |
| 21 | Education Services | 0.012 | -0.012 |
| 22 | Health and Community Services | 0.183 | -0.183 |
| 23 | Cultural and Recreational Services | 0.015 | -0.015 |
| 24 | Personal and Other Services | 0.092 | -0.092 |

The estimated coefficients for the stage 2 labour demand equations are reported in Table 8. Note that the $R^2$'s are generally higher than was the case with the net investment equations (see column 10 of Table 8). Specifically, 19 out of 24 industries now have $R^2$'s that are greater than 0.6 and 11 out of 24 industries have $R^2$'s greater than 0.8. Only two industries have negative $R^2$'s (industries 1 and 3) and only 4 industries (including industries 1 and 3) have $R^2$'s below 0.3. The sum of the coefficients associated with the distributed lag of the output terms are reported in column 9 of Table 8. All of these take values that are close to or equal to unity. This means that there will be close to a one-to-one relationship between the actual employment of labour input and the desired level of employment determined by the GL framework.

**Table 8. GL estimation results for stage 2 labour equations**

| Industry | PK | PL | TREND | OUTP | OUTP(-1) | OUTP(-2) | OUTP(-3) | $\sum out$ | RSQR |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.005 | 0.113 | 0.026 | 0.362 | 0.291 | 0.219 | 0.127 | 1.000 | -0.073 |
| 2 | 0.044 | 0.015 | 0.050 | 0.448 | 0.276 | 0.276 | 0.000 | 1.000 | 0.054 |
| 3 | 0.132 | 0.016 | 0.024 | 0.507 | 0.153 | 0.153 | 0.153 | 0.966 | -0.456 |
| 4 | 0.100 | 0.031 | 0.016 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.474 |
| 5 | 0.177 | 0.045 | 0.009 | 0.169 | 0.270 | 0.270 | 0.270 | 0.978 | 0.762 |
| 6 | 0.199 | 0.015 | 0.007 | 0.665 | 0.335 | 0.000 | 0.000 | 1.000 | 0.674 |
| 7 | 0.033 | 0.042 | 0.026 | 0.552 | 0.448 | 0.000 | 0.000 | 1.000 | 0.026 |
| 8 | 0.151 | 0.027 | 0.016 | 0.606 | 0.232 | 0.081 | 0.081 | 1.000 | 0.685 |
| 9 | 0.062 | 0.052 | 0.023 | 0.631 | 0.368 | 0.000 | 0.000 | 1.000 | 0.672 |
| 10 | 0.009 | 0.075 | 0.027 | 0.920 | 0.080 | 0.000 | 0.000 | 1.000 | 0.936 |
| 11 | 0.463 | -0.022 | -0.010 | 0.750 | 0.248 | 0.001 | 0.000 | 1.000 | 0.796 |
| 12 | 0.790 | -0.035 | 0.031 | 0.996 | 0.000 | 0.000 | 0.000 | 0.996 | 0.767 |
| 13 | 0.008 | 0.046 | 0.009 | 0.684 | 0.316 | 0.000 | 0.000 | 1.000 | 0.806 |
| 14 | 0.010 | 0.069 | 0.010 | 0.678 | 0.107 | 0.107 | 0.107 | 1.000 | 0.710 |
| 15 | 0.028 | 0.042 | -0.009 | 0.713 | 0.177 | 0.110 | 0.000 | 1.000 | 0.981 |
| 16 | 0.031 | 0.056 | 0.026 | 0.823 | 0.177 | 0.000 | 0.000 | 1.000 | 0.907 |
| 17 | 0.896 | -0.006 | 0.050 | 0.318 | 0.682 | 0.000 | 0.000 | 1.000 | 0.780 |
| 18 | 0.011 | 0.030 | 0.016 | 0.983 | 0.000 | 0.000 | 0.000 | 0.983 | 0.826 |
| 19 | 0.011 | 0.024 | -0.005 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.992 |
| 20 | 0.175 | 0.016 | 0.000 | 0.820 | 0.090 | 0.090 | 0.000 | 1.000 | 0.966 |
| 21 | 0.013 | 0.047 | 0.005 | 0.934 | 0.022 | 0.022 | 0.022 | 1.000 | 0.968 |
| 22 | 0.161 | 0.033 | 0.007 | 0.974 | 0.026 | 0.000 | 0.000 | 1.000 | 0.991 |
| 23 | 0.008 | 0.029 | -0.005 | 0.927 | 0.073 | 0.000 | 0.000 | 1.000 | 0.976 |
| 24 | 0.077 | 0.029 | -0.007 | 0.762 | 0.238 | 0.000 | 0.000 | 1.000 | 0.983 |

**(2) Autoregressive Model**

$$I_t = f\left(I_{t-1}, I_{t-2}, I_{t-3}, I_{t-4}, I_{t-5}\right),$$

where:

. $I_t$ is gross fixed capital formation (GFCF).

No single or collective constraints were imposed on the coefficients associated with the lagged values of gross fixed capital expenditure $I_t$. The objective with this specification was simply to get the best fit to the data.

The estimation results for the AR specification are reported in Table 9. This table shows the values for the intercept (INTCP), lagged investment $I_{t-i}$ (for i = 1 to 5), adjusted R-squared (RSQR), mean absolute percentage error (MAPE) and value of autocorrelation coefficient for the residuals (RHO). It is evident from inspection of Table 9 that only 5 industries have $R^2$'s that are below 0.6 and 9 industries have $R^2$'s above 0.85 – see column 11. The coefficients on the year one lag of $I_t$ are positive (and close to unity) while many coefficients are negative for the year two lag. This is indicative of strong positive serial correlation in the GFCF data at the year one lag and negative serial correlation at the year two lag. There is also evidence of alternation in the sign of the distributed lag coefficients providing evidence of the cyclical nature of the investment time series.

**Table 9. Estimation results from AR Specification**

| Industry | INTCP | $I_{t-1}$ | $I_{t-2}$ | $I_{t-3}$ | $I_{t-4}$ | $I_{t-5}$ | RSQR | MAPE | SEE | RHO |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2237.6 | 0.797 | -0.443 | 0.045 | 0.253 | -0.113 | 0.30 | 7.32 | 420.1 | 0.019 |
| 2 | 429.4 | 0.626 | 0.393 | -0.300 | 0.374 | -0.167 | 0.62 | 21.68 | 800.1 | 0.047 |
| 3 | 729.8 | 1.414 | -1.006 | 0.606 | -0.194 | 0.007 | 0.75 | 15.27 | 728.7 | 0.075 |
| 4 | -341.9 | 0.425 | 0.399 | 0.187 | 0.402 | -0.031 | 0.88 | 10.36 | 197.9 | -0.002 |
| 5 | 74.0 | 0.752 | -0.061 | 0.124 | -0.093 | -0.002 | 0.40 | 21.52 | 68.12 | 0.169 |
| 6 | 41.5 | 0.684 | 0.443 | -0.398 | -0.162 | 0.472 | 0.68 | 15.42 | 255.2 | 0.106 |
| 7 | 128.0 | 0.862 | -0.212 | 0.211 | -0.274 | 0.390 | 0.74 | 15.08 | 247.4 | 0.119 |
| 8 | 45.2 | 0.680 | -0.068 | -0.039 | -0.087 | 0.518 | 0.48 | 17.43 | 144.2 | -0.015 |
| 9 | 1001.4 | 0.995 | -0.640 | 0.267 | -0.079 | -0.156 | 0.48 | 18.96 | 395.6 | -0.029 |
| 10 | 88.3 | 0.930 | -0.321 | 0.640 | -0.401 | 0.161 | 0.79 | 20.01 | 289.3 | -0.036 |
| 11 | 117.2 | 0.548 | 0.189 | -0.292 | 0.539 | -0.503 | 0.30 | 17.59 | 43.9 | -0.155 |
| 12 | 1532.9 | 1.078 | -0.056 | -0.495 | 0.597 | -0.454 | 0.80 | 5.78 | 368.7 | -0.035 |
| 13 | 196.3 | 1.238 | -0.724 | 0.462 | -0.158 | 0.130 | 0.81 | 9.81 | 250.4 | 0.011 |
| 14 | -412.4 | 0.755 | -0.007 | 0.246 | 0.340 | -0.135 | 0.96 | 7.00 | 381.93 | -0.010 |
| 15 | 277.2 | 1.364 | -1.170 | 0.950 | -0.072 | -0.199 | 0.79 | 19.13 | 412.1 | 0.055 |
| 16 | 667.9 | 0.865 | -0.442 | 0.535 | -0.217 | 0.213 | 0.75 | 6.54 | 678.2 | 0.086 |
| 17 | -22.5 | 0.874 | -0.336 | 0.435 | 0.339 | -0.181 | 0.93 | 14.55 | 498.6 | 0.033 |
| 18 | 170.5 | 1.274 | -0.330 | 0.209 | -0.229 | 0.095 | 0.82 | 14.65 | 723.3 | -0.004 |
| 19 | 32.4 | 1.470 | -0.442 | 0.307 | -0.919 | 0.657 | 0.96 | 11.83 | 579.6 | -0.083 |
| 20 | -542.5 | 0.649 | -0.461 | 0.612 | -0.087 | 0.652 | 0.87 | 9.50 | 269.0 | 0.205 |
| 21 | 58.6 | 1.107 | -0.222 | 0.483 | -0.232 | -0.138 | 0.89 | 7.15 | 229.2 | 0.148 |
| 22 | –106.0 | 0.914 | -0.039 | 0.294 | -0.180 | 0.135 | 0.94 | 8.20 | 221.0 | 0.006 |
| 23 | -97.1 | 0.676 | 0.607 | -0.814 | -0.328 | 1.213 | 0.89 | 11.47 | 225.4 | 0.075 |
| 24 | 5.3 | 1.424 | -0.372 | 0.124 | -0.800 | 0.685 | 0.96 | 6.91 | 78.3 | -0.254 |

**(3) Accelerator Model**

$$I_t = f\left(dout_t, dout_{t-1}, dout_{t-2}, dout_{t-3}, w_t\right),$$

where:

. $dout_t$ is the first difference in real output of the purchasing industry and can be interpreted as representing demand for new capacity; and

. $w_t$ is a measure of the wear in the physical capacity of capital equipment, and, in the model, is set equal to the depreciation spill arising from the second bucket $B_2(t)$.

Note that the sum of the coefficients on the distributed lag term involving the first differences of output in the purchasing industry should be positive. The coefficient relating to $w_t$ should also be positive and theory suggests that it should be close to one since $w_t$ can be viewed as a measure of replacement investment, (see Meade 1990, pp. 119-120).

In order to obtain estimates that are consistent with theory, soft constraints were widely utilized in the estimation process. The details relating to the use of soft constraints are documented in Table 10.

In general, the soft constraints were used to ensure that the sum of the coefficients on distributed lag of variable $dout_t$ was positive but less than one in value. This would rule out the possibility of the perverse situation arising whereby investment spending would decline in times of output growth associated with periods of economic expansion. This was typically achieved using the “con” command in the G7 regression package. In certain cases, soft constraints were also employed to smooth the pattern of the lag weights of $dout_t$, thereby diminishing the extent to which they tended to jump around. This was accomplished using the “sma” command in the G7 regression package, typically with a first or second order polynomial lag structure.

These details are listed in columns 2 to 6 of Table 10, with a “*” denoting when the “sma” or “con” command (columns 2 and 3, respectively) were used in each respective industry equation. Note that the order of the polynomial lag used with the “sma” command is included in the parentheses after the “*” symbol in the sma column of Table 10, i.e. for example, “*(1)” in column 2 of Table 10 would indicate that a first order polynomial lag is used with the “sma” command. The target and actual sums (in columns 4 and 5) depict the summation target used in the “con” command and the value achieved from the softly constrained estimation process, respectively. Finally, the sum obtained when no constraints were applied in estimation is listed in column 6 of Table 10. Comparison of the values in columns 5 and 6 will give some indication of how effective the application of the soft constraints were in altering the coefficient sum associated with the lag weights on variable $dout_t$.

In some (limited) circumstances, it was necessary to apply soft constraints to get the sign of the coefficient of $w_t$ to be positive. It is evident from Table 10 (columns 7-10) that this was necessary for industries 5 and 11, respectively. The target and actual columns contain the target value for the respective coefficient that was used in the “con” command and the value achieved from the softly constrained estimation process. The last column in Table 10 contains the values for the coefficients of $w_t$ for industries 5 and 11 that were obtained when no soft constraints were imposed – in both cases, the signs on the coefficients are negative which is not consistent with theoretical reasoning, thus prompting the use of soft constraints to obtain theoretical meaningful coefficient estimates.

**Table 10. Soft Constraint Details for Estimation of Accelerator Model**

| Industry | $dout_t$ | | | | | $w_t$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | sma | con | target $\sum$ | actual $\sum$ | actual_u $\sum$ | con | target | actual | actual_u |
| 1 | | | | 0.20 | 0.20 | | | | |
| 2 | *(2) | * | 0.8 | 0.13 | -0.55 | | | | |
| 3 | | * | 0.8 | 0.41 | 0.33 | | | | |
| 4 | *(2) | * | 0.8 | 0.99 | 1.15 | | | | |
| 5 | *(1) | * | 0.3 | 0.21 | -0.21 | * | 0.15 | 0.067 | -0.24 |
| 6 | | * | 0.5 | 0.97 | 1.21 | | | | |
| 7 | *(1) | * | 0.8 | 0.99 | 0.98 | | | | |
| 8 | | * | 0.5 | 0.32 | -0.16 | | | | |
| 9 | *(2) | * | 0.9 | 0.70 | 0.66 | | | | |
| 10 | *(2) | | | 0.72 | 0.73 | | | | |
| 11 | | | | 0.35 | 0.43 | * | 0.20 | 0.082 | -0.18 |
| 12 | *(2) | * | 0.5 | 0.35 | -3.04 | | | | |
| 13 | *(2) | | | 0.32 | 0.33 | | | | |
| 14 | *(2) | | | 0.37 | 0.38 | | | | |
| 15 | *(1) | * | 0.6 | 0.83 | 2.51 | | | | |
| 16 | *(1) | * | 0.1 | 0.98 | 1.13 | | | | |
| 17 | *(1) | * | 0.9 | 0.98 | 2.54 | | | | |
| 18 | *(1) | * | 0.2 | 0.93 | 1.19 | | | | |
| 19 | *(1) | * | 0.5 | 0.87 | 2.07 | | | | |
| 20 | *(2) | | | 0.85 | 0.87 | | | | |
| 21 | | * | 0.6 | 0.39 | -0.33 | | | | |
| 22 | | * | 0.5 | 0.32 | 0.23 | | | | |
| 23 | *(1) | * | 0.7 | 0.48 | -0.20 | | | | |
| 24 | *(1) | | | 0.75 | 0.81 | | | | |

The estimation results are cited in Table 11. Some of the $R^2$'s are negative – ie. for industries 1, 5, 11 and 12. The fact that no soft constraints are used in the regression associated with industry 1 indicates that the fit for this industry is poor. All other industries have soft constraints and the negative $R^2$'s in these cases may reflect, in whole or part, the inherent difficulty in obtaining theoretically meaningful coefficient estimates – the soft constraints might have to "battle" hard against the underlying sample properties to achieve theoretically meaningful results, however, at the expense of conventional goodness of fit criteria.

However, it is apparent from examination of Table 11 that the $R^2$ for the industries mentioned above were also marginal in the case of the unconstrained regressions – for example, compare the entries in column 8 with the entries in column 12 which lists the $R^2$'s for the unconstrained regressions. It should also be noted that in all cases where the $R^2$'s are negative or positive but small in magnitude (i.e. industry 9), there is still a positive intercept and coefficients relating to $w_t$ are correctly signed – see Table 11. As such, and notwithstanding the small or negative values for $R^2$, there is no conceivable way that investment spending can behave perversely as output grows.

It is also apparent from a comparative inspection of columns 8 and 12 of Table 11 that the imposition of the soft constraints does not lead to a significant deterioration in the goodness of fit of most equations. Apart from industry 5 (which is marginal anyway), there is only a slight deterioration in the value of $R^2$ in column 8 when compared with the corresponding value in column 12 for industries 11, 12, 15 and 19, respectively.

The other notable feature of the estimation results is the magnitude of the coefficients associated with variable $w_t$. Recall that theory suggests that the coefficient should be close to one since $w_t$ can be viewed as a measure of replacement investment. However, it is evident from inspection of Table 11 (column 7) that all the estimated values for this coefficient are much smaller than unity – ranging from 0.067 for industry 5 to 0.34 for industry 23. One possible explanation for this outcome is that changes in current output could also stimulate replacement investment so that the output terms are not only capturing net investment but also some component of replacement investment. Another possible explanation is that there is high multicollinearity between changes in output and the replacement variable.

**Table 11. Estimation results from Accelerator Model**

| Industry | INTCP | $dout_t$ | $dout_{t-1}$ | $dout_{t-2}$ | $dout_{t-3}$ | $w_t$ | RSQR | MAPE | SEE | RHO | RSQR_u |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 711.4 | 0.145 | 0.083 | 0.004 | -0.031 | 0.129 | -0.03 | 9.34 | 511.3 | 0.588 | -0.03 |
| 2 | -778.5 | 0.053 | 0.084 | -0.048 | 0.040 | 0.292 | 0.62 | 22.86 | 809.0 | 0.401 | 0.64 |
| 3 | 197.1 | -0.082 | 0.115 | 0.147 | 0.232 | 0.217 | 0.44 | 24.54 | 1097.0 | 0.663 | 0.44 |
| 4 | -684.1 | 0.339 | 0.221 | 0.257 | 0.177 | 0.305 | 0.83 | 14.82 | 240.3 | 0.596 | 0.84 |
| 5 | 148.4 | -0.012 | 0.071 | 0.094 | 0.060 | 0.067 | -0.34 | 40.43 | 102.0 | 0.784 | 0.04 |
| 6 | -820.0 | 0.387 | 0.244 | 0.238 | 0.100 | 0.331 | 0.81 | 13.35 | 199.1 | 0.124 | 0.81 |
| 7 | -267.8 | 0.384 | 0.371 | 0.231 | 0.009 | 0.245 | 0.73 | 16.66 | 250.6 | 0.513 | 0.73 |
| 8 | -113.1 | 0.059 | 0.023 | 0.103 | 0.135 | 0.258 | 0.38 | 18.37 | 156.7 | 0.426 | 0.40 |
| 9 | 820.4 | -0.236 | 0.184 | 0.390 | 0.367 | 0.078 | 0.05 | 27.37 | 533.8 | 0.663 | 0.05 |
| 10 | -227.6 | 0.285 | 0.144 | 0.161 | 0.130 | 0.293 | 0.82 | 24.83 | 267.5 | 0.419 | 0.82 |
| 11 | 110.1 | 0.118 | 0.135 | 0.105 | -0.005 | 0.082 | -0.03 | 19.08 | 53.4 | 0.489 | 0.08 |
| 12 | 1648.3 | 0.304 | 0.180 | -0.034 | -0.104 | 0.073 | -0.14 | 15.93 | 885.7 | 0.847 | -0.06 |
| 13 | 552.8 | 0.083 | 0.135 | 0.051 | 0.054 | 0.145 | 0.78 | 12.01 | 274.8 | 0.614 | 0.78 |
| 14 | -1002.9 | 0.075 | 0.134 | 0.090 | 0.076 | 0.252 | 0.94 | 7.23 | 439.3 | 0.565 | 0.94 |
| 15 | -32.9 | 0.329 | 0.538 | 0.183 | -0.218 | 0.223 | 0.69 | 27.52 | 499.1 | 0.542 | 0.74 |
| 16 | 534.7 | 0.236 | 0.413 | 0.252 | 0.081 | 0.124 | 0.76 | 6.61 | 665.3 | 0.342 | 0.76 |
| 17 | -814.7 | 0.097 | 0.334 | 0.371 | 0.175 | 0.300 | 0.94 | 13.71 | 459.5 | 0.272 | 0.94 |
| 18 | -47.6 | 0.187 | 0.437 | 0.505 | -0.199 | 0.299 | 0.89 | 15.08 | 563.7 | 0.618 | 0.90 |
| 19 | -40.1 | 0.210 | 0.315 | 0.207 | 0.136 | 0.237 | 0.91 | 16.34 | 892.5 | 0.795 | 0.96 |
| 20 | -2650.6 | 0.217 | 0.239 | 0.195 | 0.198 | 0.261 | 0.83 | 10.21 | 310.5 | 0.477 | 0.83 |
| 21 | -2202.4 | 0.340 | 0.160 | 0.013 | -0.118 | 0.245 | 0.43 | 17.27 | 513.8 | 0.890 | 0.46 |
| 22 | -920.5 | 0.108 | 0.092 | 0.169 | -0.053 | 0.254 | 0.89 | 11.58 | 290.2 | 0.734 | 0.89 |
| 23 | -177.4 | -0.062 | 0.126 | 0.252 | 0.166 | 0.340 | 0.80 | 14.72 | 313.1 | 0.473 | 0.80 |
| 24 | -221.3 | 0.105 | 0.144 | 0.259 | 0.241 | 0.260 | 0.91 | 11.94 | 120.0 | 0.774 | 0.91 |

**(4) Jorgenson Cobb-Douglas Model**

$$I_t = f\left(\Delta\left[\frac{pQ}{c}\right]_t, \Delta\left[\frac{pQ}{c}\right]_{t-1}, \Delta\left[\frac{pQ}{c}\right]_{t-2}, \Delta\left[\frac{pQ}{c}\right]_{t-3}, K_{t-1}\right),$$

where:

. $\Delta$ denotes the first difference operator;
. $p$ is the price of output in the purchasing industry;
. $Q$ is real output of the purchasing industry;
. $c$ is the user cost of capital; and
. $K_t$ is the productive capital stock of the purchasing industry.

The distributed lag term involving the composite price-output term $\left[\frac{pQ}{c}\right]_t$ is employed to account for a postulated lag between changes in the desired capital stock and actual investment. The composite price-output variable ensures that the distributed lag pattern on real output $Q$ and the relative user cost of capital $\left[\frac{p}{c}\right]$ are the same. The last term involving the lagged productive capital stock is used to capture replacement investment. The desired stock of capital is given by:

$$K_t^* = \alpha * \left[\frac{pQ}{c}\right]_t$$

where $\alpha$ is the share parameter of capital in the conventional Cobb-Douglas production function, (see Meade 1990, pp. 120-123).

The sum of the coefficients on the distributed lag term involving the differenced composite price-output term $\Delta\left[\frac{pQ}{c}\right]_t$ are expected to sum to a positive value because investment should respond positively to both changes in real output and the price of output, and negatively to increases in the user cost of capital. The coefficient on the lagged capital stock is expected to be small and positive and can be interpreted as an estimate of geometric depreciation rate (Meade 1990, pp. 121-122).

Replacement investment equals $\delta K_{t-1}$ where $\delta$ is the estimated coefficient of $K_{t-1}$ and net investment is given by the difference between GFCF ($I_t$) and replacement investment.

Soft constraints were again imposed using the g7 “con” command to ensure that the sum of the coefficients of the distributed lag of the differenced composite price-output variable $\Delta\left[\frac{pQ}{c}\right]_t$ was positive. In some cases, soft constraints were also imposed using the “sma” command to smooth the pattern of the lag weights of $\Delta\left[\frac{pQ}{c}\right]_t$. These details are listed in columns 2 to 6 of Table 12 using the same notation that was introduced in the discussion of Table 10 in the previous section.

In some circumstances, it was also necessary to apply soft constraints to get the sign of the coefficient of $K_{t-1}$ to be positive. It is apparent from Table 12 (columns 7-10) that this was necessary for industries 5, 9 and 11, respectively. Note that the last column of Table 12 contains the values for the coefficients of $K_{t-1}$ for industries 5, 9 and 11 that were obtained when no soft constraints were imposed. The signs of the coefficients in the last column are negative which is not consistent with theory and can be contrasted with the correctly signed coefficients obtained after application of the soft constraints - see column 9 of Table 12.

**Table 12. Soft Constraint Details for Jorgenson Cobb-Douglas Model**

| **Industry** | $\Delta\left[\frac{pQ}{c}\right]_t$ | | | | | $K_{t-1}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **sma** | **con** | **target** $\sum$ | **actual** $\sum$ | **actual_u** $\sum$ | **con** | **target** | **actual** | **actual_u** |
| 1 | | | | 0.11 | 0.11 | | | | |
| 2 | | * | 0.3 | 0.04 | -0.04 | | | | |
| 3 | | * | 0.4 | 0.24 | 0.15 | | | | |
| 4 | | | | 0.24 | 0.24 | | | | |
| 5 | | | | 0.11 | 0.08 | * | 0.10 | 0.04 | -0.002 |
| 6 | *(2) | | | 0.28 | 0.28 | | | | |
| 7 | | * | 0.2 | 0.07 | 0.98 | | | | |
| 8 | *(2) | * | 0.2 | 0.16 | -0.001 | | | | |
| 9 | *(2) | | | 0.18 | 0.07 | * | 0.10 | 0.02 | -0.004 |
| 10 | | | | 0.10 | 0.10 | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | | | | 0.10 | 0.11 | * | 0.15 | 0.02 | -0.038 |
| 12 | | | | 0.21 | 0.21 | | | | |
| 13 | | * | 0.2 | 0.03 | 0.03 | | | | |
| 14 | | | | 0.09 | 0.09 | | | | |
| 15 | | | | 0.20 | 0.20 | | | | |
| 16 | | | | 0.10 | 0.10 | | | | |
| 17 | | * | 0.2 | 0.18 | -0.25 | | | | |
| 18 | | | | 0.28 | 0.28 | | | | |
| 19 | | | | 0.20 | 0.20 | | | | |
| 20 | | | | 0.07 | 0.07 | | | | |
| 21 | | | | 0.16 | 0.16 | | | | |
| 22 | | | | 0.07 | 0.07 | | | | |
| 23 | *(1) | * | 0.2 | 0.17 | -0.06 | | | | |
| 24 | *(1) | | | 0.19 | 0.19 | | | | |

The estimation results for the Jorgenson Cobb-Douglas model are reported in Table 13. Some of the $R^2$'s are negative – ie. for industries 5 and 9, while a small positive value for $R^2$ is obtained for industry 12 – see column 8 of Table 13. It is apparent, however, from column 12 of Table 13 that the $R^2$'s for these industries are also very marginal in the case of the unconstrained regressions. Table 13 also shows that 17 out of 24 industries have $R^2$'s greater than 0.6 and 9 industries have $R^2$ greater than 0.8.

A comparative inspection of columns 8 and 12 of Table 13 indicates that the imposition of the soft constraints did not lead to a significant deterioration in the goodness of fit in any industry. Poor goodness of fit outcomes given by low or negative $R^2$'s could be attributed to the model's inadequacy in fitting the data and was not related to the imposition of soft constraints in estimation.

**Table 13. Estimation results from Jorgenson Cobb-Douglas Model**

| **Industry** | **INTCP** | $\Delta\left[\frac{pQ}{c}\right]_t$ | $\Delta\left[\frac{pQ}{c}\right]_{t-1}$ | $\Delta\left[\frac{pQ}{c}\right]_{t-2}$ | $\Delta\left[\frac{pQ}{c}\right]_{t-3}$ | $K_{t-1}$ | **RSQR** | **MAPE** | **SEE** | **RHO** | **RSQRc** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1387.0 | 0.044 | 0.046 | 0.022 | 0.002 | 0.050 | 0.34 | 6.97 | 409.4 | 0.548 | 0.34 |
| 2 | -610.8 | 0.004 | 0.036 | 0.024 | -0.025 | 0.133 | 0.60 | 24.05 | 828.0 | 0.396 | 0.61 |
| 3 | 331.5 | 0.030 | 0.037 | 0.086 | 0.085 | 0.096 | 0.42 | 26.95 | 1117.8 | 0.679 | 0.43 |
| 4 | -33.5 | 0.085 | 0.048 | 0.067 | 0.038 | 0.088 | 0.89 | 11.19 | 197.2 | 0.243 | 0.89 |
| 5 | 118.5 | 0.024 | 0.043 | 0.021 | 0.018 | 0.039 | -0.12 | 35.21 | 93.4 | 0.728 | -0.09 |
| 6 | -11.5 | 0.096 | 0.056 | 0.082 | 0.047 | 0.078 | 0.77 | 14.88 | 215.4 | 0.013 | 0.77 |
| 7 | -8.4 | 0.050 | 0.001 | 0.026 | -0.008 | 0.103 | 0.66 | 19.49 | 280.0 | 0.491 | 0.67 |
| 8 | 23.6 | 0.048 | 0.043 | 0.029 | 0.043 | 0.090 | 0.35 | 19.92 | 160.4 | 0.376 | 0.36 |
| 9 | 1121.5 | 0.000 | 0.046 | 0.056 | 0.076 | 0.017 | -0.02 | 24.88 | 553.1 | 0.647 | -0.02 |
| 10 | -98.7 | 0.046 | 0.022 | 0.014 | 0.023 | 0.121 | 0.77 | 24.69 | 302.3 | 0.498 | 0.77 |
| 11 | 131.8 | 0.027 | 0.035 | 0.035 | 0.007 | 0.025 | 0.31 | 17.78 | 43.8 | 0.364 | 0.33 |
| 12 | 2889.2 | 0.071 | 0.056 | 0.039 | 0.041 | 0.016 | 0.07 | 14.36 | 802.1 | 0.762 | 0.06 |
| 13 | 624.9 | 0.018 | 0.015 | 0.011 | -0.013 | 0.066 | 0.77 | 10.84 | 278.2 | 0.628 | 0.77 |
| 14 | 19.0 | 0.020 | 0.019 | 0.026 | 0.010 | 0.090 | 0.96 | 6.27 | 368.9 | 0.525 | 0.96 |
| 15 | 424.3 | 0.038 | 0.060 | 0.133 | -0.026 | 0.065 | 0.63 | 29.15 | 542.9 | 0.566 | 0.63 |
| 16 | 1081.2 | 0.004 | 0.051 | 0.034 | 0.007 | 0.057 | 0.83 | 5.56 | 565.3 | 0.508 | 0.83 |
| 17 | -582.3 | 0.027 | 0.037 | 0.041 | 0.076 | 0.136 | 0.91 | 18.15 | 570.0 | 0.322 | 0.93 |
| 18 | 1229.8 | 0.053 | 0.117 | 0.074 | 0.036 | 0.044 | 0.84 | 17.25 | 680.7 | 0.607 | 0.84 |
| 19 | 1355.4 | 0.054 | 0.058 | 0.046 | 0.040 | 0.052 | 0.90 | 20.19 | 921.0 | 0.707 | 0.90 |
| 20 | -781.8 | -0.009 | 0.027 | 0.012 | 0.038 | 0.075 | 0.93 | 6.55 | 184.7 | 0.517 | 0.94 |
| 21 | 1055.8 | 0.039 | 0.039 | 0.037 | 0.044 | 0.023 | 0.77 | 9.63 | 323.1 | 0.251 | 0.77 |
| 22 | -231.0 | 0.014 | 0.028 | 0.018 | 0.012 | 0.087 | 0.93 | 9.16 | 231.5 | 0.403 | 0.93 |
| 23 | 36.4 | 0.032 | 0.040 | 0.016 | 0.079 | 0.125 | 0.77 | 18.16 | 335.1 | 0.542 | 0.80 |
| 24 | 297.2 | 0.049 | 0.055 | 0.060 | 0.031 | 0.026 | 0.93 | 10.72 | 107.3 | 0.607 | 0.93 |

The values of the coefficient on the lagged capital stock variable $K_{t-1}$ seem acceptable and, on the whole, the estimated values fall reasonably close to the values calculated assuming geometric depreciation and given the average service lives used in the user cost of capital calculations. These results are documented in Table 14.

**Table 14. Comparison of estimated depreciation rates with calculated depreciation rates[5]**

| Industry | Calculated | Estimated | L |
|---|---|---|---|
| 1 | 0.069 | 0.050 | 13.5 |
| 2 | 0.080 | 0.133 | 11.5 |
| 3 | 0.080 | 0.096 | 11.5 |
| 4 | 0.075 | 0.088 | 12.4 |
| 5 | 0.075 | 0.039 | 12.4 |
| 6 | 0.075 | 0.078 | 12.4 |
| 7 | 0.075 | 0.103 | 12.4 |
| 8 | 0.075 | 0.090 | 12.4 |
| 9 | 0.075 | 0.017 | 12.4 |
| 10 | 0.075 | 0.121 | 12.4 |
| 11 | 0.075 | 0.025 | 12.4 |
| 12 | 0.045 | 0.016 | 21.2 |
| 13 | 0.074 | 0.066 | 12.6 |
| 14 | 0.067 | 0.090 | 14.0 |
| 15 | 0.071 | 0.065 | 13.0 |
| 16 | 0.051 | 0.057 | 18.5 |
| 17 | 0.075 | 0.136 | 12.4 |
| 18 | 0.130 | 0.044 | 6.7 |
| 19 | 0.082 | 0.052 | 11.2 |
| 20 | 0.043 | 0.075 | 22.4 |
| 21 | 0.049 | 0.023 | 19.4 |
| 22 | 0.058 | 0.087 | 16.2 |
| 23 | 0.092 | 0.125 | 9.9 |
| 24 | 0.069 | 0.026 | 13.4 |

**(5) Ranking of Autoregressive, Accelerator and Jorgenson Cobb-Douglas Models**

A comparative ranking of the 3 models is listed in Table 15. These models were ranked by two criteria – the adjusted R-square ( $R^2$ ) and the mean absolute percentage error (MAPE). The "best" model for each criteria is given a ranking of 1, followed by the next "best" model which is given a ranking of 2, followed by the remaining model which is given a ranking of 3. For the $R^2$ criteria, "best" is taken to be the model with the highest value for $R^2$, while for the MAPE statistic, "best" is taken to be the model with the smallest MAPE value.

**Table 15. Comparative rankings of AR (AR), Accelerator (AC) and Jorgenson Cobb Douglas (JCD) models**

| Industry | Ranking by $R^2$ | | | Ranking by MAPE | | |
|---|---|---|---|---|---|---|
| | AR | AC | JCD | AR | AR | JCD |
| 1 | 2 | 3 | 1 | 2 | 3 | 1 |
| 2 | 1 | 2 | 3 | 1 | 2 | 3 |
| 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| 4 | 2 | 3 | 1 | 1 | 3 | 2 |
| 5 | 1 | 3 | 2 | 1 | 3 | 2 |
| 6 | 3 | 1 | 2 | 3 | 1 | 2 |
| 7 | 1 | 2 | 3 | 1 | 2 | 3 |

[5] Assuming geometric depreciation, column 2 of Table 14 is calculated as:

$\delta = \left( \frac{1}{(1+L)} \right)$, where L is assumed average service life of capital goods in each purchasing industry which is listed in column 4 of Table 14, (see Meade 1990, p. 169).

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | 1 | 2 | 3 | 1 | 2 | 3 |
| 9 | 1 | 2 | 3 | 1 | 3 | 2 |
| 10 | 2 | 1 | 3 | 1 | 3 | 2 |
| 11 | 2 | 3 | 1 | 1 | 3 | 2 |
| 12 | 1 | 3 | 2 | 1 | 3 | 2 |
| 13 | 1 | 2 | 3 | 1 | 3 | 2 |
| 14 | 1 | 3 | 2 | 2 | 3 | 1 |
| 15 | 1 | 2 | 3 | 1 | 2 | 3 |
| 16 | 3 | 2 | 1 | 2 | 3 | 1 |
| 17 | 2 | 1 | 3 | 2 | 1 | 3 |
| 18 | 3 | 1 | 2 | 1 | 2 | 3 |
| 19 | 1 | 2 | 3 | 1 | 2 | 3 |
| 20 | 2 | 3 | 1 | 2 | 3 | 1 |
| 21 | 1 | 3 | 2 | 1 | 3 | 2 |
| 22 | 1 | 3 | 2 | 1 | 3 | 2 |
| 23 | 1 | 2 | 3 | 1 | 2 | 3 |
| 24 | 1 | 3 | 2 | 1 | 3 | 2 |

In Table 15, the ranking schemes for $R^2$ are listed in columns 2-4 with AR denoting the comparative ranking of the AR model for each industry and columns 3 and 4 similarly depicting the comparative ranking of the Accelerator (AC) and Jorgenson Cobb-Douglas (JCD) models, respectively. It is apparent from these columns that the AR model is the best model (having a ranking of 1) according to the $R^2$ criteria for 15 of the 24 purchasing industries. The AC model is the best model for 4 of 24 industries while the JCD model is the best model for 5 of the 24 industries. In terms of second best models (having a ranking of 2), the AR model is second best in 6 out of 24 industries, the AC model is second best in 10 of 24 industries while the JCD model is second best in 8 of 24 industries. Finally, the AR model is the worst model (having a ranking of 3) in 3 out of 24 industries, while the AC model is the worst model in 10 of 24 industries and the JCD model is the worst model in 11 out of 24 industries.

Clearly, the best model according to the $R^2$ criteria is the AR model. This result is not surprising because this was the precise reason why this model was adopted. The AC and JCD model’s overall rankings are very similar.

The ranking schemes for the MAPE criteria are listed in columns 5-7 of Table 15. The AR model is the best model for 18 out of 24 industries while the AC model is the best model for 2 out of 24 industries and the JCD model is the best model for 4 of the 24 industries. In terms of second best models, the AR model is second best in 5 of 24 industries, the AC model is second best in 8 of 24 industries while the JCD model is second best in 11 out of 24 industries. Finally, the AR model is the worst model in 1 of 24 industries, while the AC model is the worst model in 14 of 24 industries and the JCD model is the worst model in 9 out of 24 industries. Once again, the best model according to the MAPE criteria is the AR model. However, the JCD model seems to have a more recognizable advantage over the AC model according to the MAPE criteria.

The choice of “best” investment model will be ultimately determined by the stability and forecast performance of the models when run as part of the complete model. Therefore, while the results listed in Table 15 are insightful to some extent, the conclusion about “best” model does not represent our final determination on this issue. For one matter, the AR model has essentially no economic content or motivation justifying its use. In another context, the ability to fit the data does not preclude the possibility of perverse simulation outcomes, especially as the system is pushed away from its underlying sample path or run simultaneously as a component within a larger non-linear IO-econometric model. This was the principal reason why soft constraints were employed to achieve outcomes that were consistent with theoretical reasoning.

Constrained specifications are likely to engender a greater degree of stability and superior forecast performance when viewed from the context of the general simulation properties of the complete model. Finally, our ultimate interest is in providing economic explanations for modeling investment spending.

It is for this reason that we are principally interested in investigating how the economic models – namely the Generalised Leontief (GL), AC and JCD models - work within the context of the broader IO-Econometric model.

**DWELLING INVESTMENT EXPENDITURE**

**Key Endogenous variables:**

. Productive Capital Stock for Ownership of Dwellings Industry
. Depreciation spill associated with Productive Capital Stock
. GFCF (CVM) for Ownership of Dwellings Industry

**Discussion**

Dwelling investment refers to investment expenditure (purchases) by industry 25, "Ownership of Dwellings". The productive capital stock for this sector is determined from the two-bucket scheme associated with a second order pascal lag distribution. The user cost of capital for this sector is also determined within the model using the same formula that was used for non-dwelling investment.

In attempts to model dwelling investment, a number of different models have been investigated:

**(1) AR Specification**

$$I_t = \beta_0 + \beta_1 * I_{t-1} + \beta_2 * I_{t-2} + \beta_3 * I_{t-3} + \beta_4 * I_{t-4} + \beta_5 * I_{t-5} .$$

The coefficient estimates for the AR model were estimated by OLS and are:

**Dwelling Investment – AR specification**

| Parameter | Coefficient Estimate | t-values |
|---|---|---|
| $\beta_0$ | 481.559 | 0.22 |
| $\beta_1$ | 0.513 | 2.96 |
| $\beta_2$ | -0.233 | -0.66 |
| $\beta_3$ | 0.017 | 0.04 |
| $\beta_4$ | 0.456 | 1.20 |
| $\beta_5$ | 0.315 | 0.99 |

$R^2$ = 0.81, MAPE = 7.03, SEE = 1900.27, RHO = 0.09.

**(2) Accelerator Model**

$$I_t = \beta_0 + \beta_1 * dout_t + \beta_2 * dout_{t-1} + \beta_3 * dout_{t-2} + \beta_4 * dout_{t-3} + \beta_5 * w_t , \qquad (+)$$

where the regressor variables $dout_t$ and $w_t$ are the same variables and have the same meanings as discussed previously in relation to accelerator models of non-dwelling investment.

The coefficient estimates for the Accelerator model (estimated by OLS) are:

**Dwelling Investment – Accelerator specification**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | 1506.749 |
| $\beta_1$ | 0.578 |

| | |
|---|---|
| $\beta_2$ | 0.354 |
| $\beta_3$ | -0.306 |
| $\beta_4$ | 0.273 |
| $\beta_5$ | 0.134 |

$R^2$ = 0.78, MAPE = 7.86, SEE = 2073.11, RHO = 0.29.

The t statistics are not listed above because soft constraints were employed in the estimation of the above equation and the soft constraints will invalidate the conventional interpretation of t statistics.

**(3) Jorgenson Cobb-Douglas Model**

$$I_t = \beta_0 + \beta_1 * \Delta\left[\frac{pQ}{c}\right]_t + \beta_2 * \Delta\left[\frac{pQ}{c}\right]_{t-1} + \beta_3 * \Delta\left[\frac{pQ}{c}\right]_{t-2} + \beta_4 * \Delta\left[\frac{pQ}{c}\right]_{t-3} + \beta_5 * K_{t-1}$$

where the regressor variables have the same definitions and meanings as discussed previously in conjunction with Jorgenson-Cobb Douglas models of non-dwelling investment.

The coefficient estimates for the Jorgenson Cobb-Douglas model (estimated by OLS) are:

**Dwelling Investment – Jorgenson Cobb-Douglas specification**

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\beta_0$ | 3335.960 | 2.08 |
| $\beta_1$ | 0.017 | 0.60 |
| $\beta_2$ | 0.116 | 4.35 |
| $\beta_3$ | 0.080 | 2.76 |
| $\beta_4$ | 0.025 | 0.77 |
| $\beta_5$ | 0.053[6] | 8.32 |

$R^2$ = 0.90, MAPE = 5.05, SEE = 1416.54, RHO = 0.15.

**(4) IDLIFT Type equation**

This equation is substantially based on the residential construction specification employed in the U.S. IDLIFT model (see Meade 2000, 2001 and Horst 2002). Specifically, the equation is estimated in a per-capita form and based on household gross disposable income per capita, the mortgage rate and a demographic variable that is likely to capture the impact of changes in the relative size of the prime age cohort most closely associated with first home buyers. The form of the equation is as follows:

$$I_t^{\wedge} = \beta_0 + \underset{(+)}{\beta_1 * pchdi_t} + \underset{(+)}{\beta_2 * dpchdi_t} + \underset{(+)}{\beta_3 * dpchdi_{t-1}} + \underset{(+)}{\beta_4 * hhper_t} + \underset{(-)}{\beta_5 * rcmor_t},$$

where:

- $I_t^{\wedge}$ is per capita dwelling investment;
- $pchdi_t$ is real per capita household gross disposable income;

[6] Note that the geometric depreciation rate equals [1/(1+av.life) = 1/(1+19.8) = 0.048] which is quite close to the estimated value obtained for $\beta_5$ of 0.053.

- . $dpchdi_t$ is the first difference in $pchdi_t$;
- . $hhper_t$ is the ratio of 25 to 34 age cohort of the estimated resident population to the total estimated resident population; and
- . $rcmor_t$ is the mortgage interest rate.

The signs on the coefficients have reasonably straight-forward interpretations. We would expect dwelling investment to grow with positive growth in real household gross disposable income and to decline as mortgage interest rates rise. The population ratio was included to capture the positive impact that we would expect to arise as more individuals enter the 25 to 34 age cohort – the prime age cohort associated with first home purchases.

The coefficient estimates for the IDLIFT type model (estimated by OLS) are:

**Dwelling Investment – IDLIFT type specification**

| Parameter | Coefficient Estimate | t-values |
|---|---|---|
| $\beta_0$ | -2271.173 | -2.19 |
| $\beta_1$ | 0.050 | 2.68 |
| $\beta_2$ | 0.184 | 2.44 |
| $\beta_3$ | 0.041 | 0.59 |
| $\beta_4$ | 172.956 | 2.53 |
| $\beta_5$ | -35.565 | -2.61 |

$R^2$ = 0.64, MAPE = 5.78, SEE = 99.44, RHO = -0.01.

**(5) TRYM and AEM Type specifications**[7]

The following two specifications employ the concept of Q ratio as a major determinant of dwelling investment. This ratio can be defined as the ratio of the expected rate of return from an extra unit of dwelling capital to the required rate of return of that investment assuming profit maximising behaviour. It can be viewed as an index variable that will equal one when dwelling investment is sufficient to maintain the desired growth in the stock of dwellings. If the Q ratio is greater than one, households would be expected to invest in dwellings and if it is less than one, households would be expected to reduce their investment in dwellings.

The expected rate of return on marginal dwelling investment is linked to the tax adjusted relative price of additional investment multiplied by rental services produced by this additional unit of dwelling investment. The opportunity cost of an additional unit of dwelling investment will be linked to the long run real interest rate plus the rate of depreciation on the stock of dwellings.

It should be noted that one current limitation with the treatment of the Q ratio in the model is that the Q ratio is exogenous, being obtained from the TRYM modeller’s data base. Furthermore, the projection of the Q ratio to 2030 was obtained by simulating the TRYM model under their baseline scenario. Clearly, because the Q ratio represents the relationship between the expected and required return on an additional unit of dwelling investment, it should be an endogenous variable. This is one task that will be investigated in future development of the model.

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[7] Documentation of the TRYM model is contained in Commonwealth Treasury (1996). The AEM specification was outlined in Murphy (1988, pp. 178, 183, 1992 [Section G]) and Wild (1995, pp. 88-91,427).

In the short run, dwelling investment is postulated to be a function of contemporaneous and lagged values of the Q ratio. The importance of the distributed lag structure is consistent with the notion of lags in households' response to changes in economic conditions associated with the time taken in planning dwelling investment and in obtaining approval to commence construction. Past dwelling investment is also included as an explanatory variable and helps capture the "lumpy" and cyclical nature of dwelling investment. A homogeneity constraint is placed on the distributed lag structure associated with lagged values of the dependent variable and can be interpreted as giving the long run equilibrium level of dwelling investment.

Heuristically, the TRYM specification is:

$$I_t^{\vee} = (GR+\lambda)*(1-\beta_0-\beta_1)+\beta_0*I_{t-1}^{\vee}+\beta_1*I_{t-2}^{\vee}+\underset{(+)}{\beta_2}*((Qd_t-1)+(Qd_{t-1}-1)+(Qd_{t-2}-1))+\underset{(-)}{\beta_3}*ratd_t$$

while the AEM specification is:

$$I_t^{\vee} = \beta_0+(GR^*+\lambda-1)*(1-\beta_1-\beta_2)+\beta_1*I_{t-1}^{\vee}+\beta_2*I_{t-2}^{\vee}+\underset{(+)}{\beta_3}*((Qd_t-1)+(Qd_{t-1}-1)+(Qd_{t-2}-1))+\underset{(-)}{\beta_4}*ratd_t$$

where:

. $I_t^{\vee} = \dfrac{I_t}{K_{t-1}}$;
. $GR$ is the underlying real growth rate of the economy – calculated as the percentage growth in potential GDP;
. $GR^* = 1+GR$;
. $\lambda$ is the rate of depreciation for dwellings – set equal to the rate used to calculate the productive capital stock;
. $Qd_t$ is (exogenous) Q ratio for dwellings investment; and
. $ratd_t$ is a variable denoting excessive tightness in the financial sector and is defined as the short term interest rate minus the long term interest rate.

The expected signs on coefficients $\beta_2$ and $\beta_3$ (or $\beta_3$ and $\beta_4$ in the AEM specification) are positive and negative respectively. Specifically, if $Qd_t$ is greater than one, then according to the Q theory, households will have an incentive to invest in additional units of dwellings investment thus producing the positive sign. Similarly, if $ratd_t$ is positive, this means that short term interest rates are greater than long run rates, pointing currently to the pursuit of tight monetary policy, thereby increasing the opportunity cost (or required rate of return) of additional dwelling investment relative to the expected rate of return, thus reducing dwelling investment. Furthermore, the sum of coefficients $\beta_0$ and $\beta_1$ (coefficients $\beta_1$ and $\beta_2$ in the AEM specification) are expected to be positive and less than 1. The long run equilibrium level of dwelling investment is given (for the TRYM specification) by the expression:

$$(GR+\lambda)*(1-\beta_0-\beta_1)+\beta_0*I_{t-1}^{\vee}+\beta_1*I_{t-2}^{\vee}.$$

The coefficient estimates for the TRYM and AEM equations (which is estimated by the simplex based non-linear method in the g7 package) are:

**Dwelling Investment – TRYM specification**
**Parameter Coefficient Estimate**

| | |
|---|---|
| $\beta_0$ | 0.736 |
| $\beta_1$ | 0.170 |
| $\beta_2$ | 0.1e-5 |
| $\beta_3$ | -0.002 |

**Dwelling Investment – AEM specification**

| Parameter | Coefficient Estimate |
|---|---|
| $\beta_0$ | -0.010 |
| $\beta_1$ | 0.363 |
| $\beta_2$ | -0.140 |
| $\beta_3$ | 0.6e-5 |
| $\beta_4$ | -0.002 |

Once again, the t statistics are not listed above because soft constraints were employed in the estimation of the above equations which serves to invalidate the conventional interpretation of t statistics. Specifically, soft constraints were employed to ensure that the coefficients on $Qd_t$ (coefficients $\beta_2$ and $\beta_3$ in the TRYM and AEM models, respectively) were positive.

One investment item is treated as an exogenous variable - ownership transfer expenses (alternatively termed) real estate transfer expenses. Ownership transfer costs is combined with dwelling and non-dwelling investment to obtain gross fixed capital expenditure.

Non-dwelling and dwelling investment are combined and included in bridging operations to determine both investment prices and the disposition (sale) of industry output to gross fixed capital formation.

## (C) POTENTIAL OUTPUT

**Key Endogenous Variables:**

. Potential GDP
. GDP gap

**Discussion**

In the previous discussion of both household savings and dwelling investment, variables termed GDPGAP and $GR$ (defined as the underlying real growth rate of the economy – calculated as the percentage growth in potential GDP) were employed respectively. These two variables are related to the concept of Potential GDP. Potential GDP is used in the model as a measure of overall tightness of the economy. The GDP gap is an index variable that rises above 100 when the economy is "tight" – that is, when actual GDP exceeds potential GDP. Specifically, the GDP gap is defined as (GDP/GDP_POT)*100, where GDP_POT denotes potential GDP. The concept of potential GDP refers to the level of GDP at which the economy is running at its capacity with the capacity, in turn, being determined by labour force growth, labour participation and labour productivity, (see Meade 2001, pp.13-14).

In estimating the potential GDP equation, we use the following functional form:

$$\log(gdp_t) = \beta_0 + \underset{(=1)}{\beta_1} * \left(\log(smprd_t) + \log(smlfc_t) + \log(smhrs_t)\right)$$

where coefficient $\beta_1$ is constrained to unity, and where:

. $gdp_t$ is gross domestic product;
. $smprd_t$ is a five year moving average of aggregate labour productivity;
. $smlfc_t$ is smoothed labour force, calculated as a five year moving average of the labour force participation rate multiplied by the adult population; and
. $smhrs_t$ is a five year moving average of aggregate hours worked.

The above equation was estimated using the OLS "r" command in the g7 package. The coefficient estimates were:

| **Parameter** | **Coefficient Estimate** |
|---|---|
| $\beta_0$ | -6.940 |
| $\beta_1$ | 1.000 |

$R^2$ = 0.99, MAPE = 0.13, SEE = 0.02, RHO = 0.61.

The t statistics are not included because coefficient $\beta_1$ was softly constrained to have a value equal to one. This operation will invalidate the conventional interpretation of t statistics. Potential GDP is set equal to the predicted values arising from the above regression equation.

# (D) LABOUR MARKET MODULE

## LABOUR PRODUCTIVITY, AVERAGE HOURS WORKED AND EMPLOYMENT

In the national model, two specific measures of industry employment are used: total hours worked and employed persons. The initial employment measure used is total hours worked. In broad terms, two particular approaches have been adopted to model total hours worked. First, the definition of labour input used in the GL model is total hours worked. In the second approach, total hours worked is derived from estimated industry labour productivity equations. Specifically, total hours worked by industry can be obtained by dividing industry output by industry labour productivity.

### (1) Industry Labour Productivity Equations[8]

For each industry, labour productivity equations can be written functionally as:

$$\ln\left(\frac{Q_t}{h_t}\right) = f\left(qup_t, qdown_t, dq_t, pqag_t, t\right)$$

where:

- $Q_t =$ industry output;
- $h_t =$ total hours worked by industry;
- $t =$ a linear time trend;
- $qup_t = dq_t$, when $dq_t$ is positive, 0 otherwise;
- $qdown_t = dq_t$, when $dq_t$ is negative, 0 otherwise;
- $dq_t = \ln(Q_t) - \ln(qpeak_{t-1})$;
- $qpeak_t = Q_t$, if $Q_t > qpeak_{t-1}(1-spill)$, otherwise $= qpeak_{t-1}(1-spill)$;
- $spill =$ depreciation rate of capacity, both in the sense of capital and “hoarded” labour; and
- $pqag_t =$ price index of industry output.

The time trend term $t$ is used to capture observed changes in the rate of labour productivity growth arising in various industries. The $qup_t$ and $qdown_t$ terms capture the increase in labour productivity that is observed in periods of increasing output, and vice-versa.

This latter phenomenon of pro-cyclical labour productivity is often associated with “labour hoarding”, which arises when firms retain trained workers in periods of downturn. Specifically, this effect is based on the proposition that there is a lag in the movements of hours worked behind output, partially due to the fact that employers tend to "hoard" workers in a downturn, until it becomes obvious that the downturn is going to be long and deep. Conversely, when there is an upturn, firms tend to wait a while before hiring new labour, and work the existing workforce more intensively. When output expands again, they put the hoarded labour back to work before hiring new workers.

For this phenomenon to hold, the ratio (Q/h) should increase relative to trend in an upturn (i.e. Q moves faster than h) and decrease in a downturn. While

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[8] Once again, both the discussion and motivation of the material in this section was greatly advanced by discussions held with Douglas Meade. Also see Meade (2000, pp.8-9) and Meade (2001, pp.12-13 ).

variables $qup_t$ and $qdown_t$ are not exactly the same as upturns or downturns in the economy, they can reasonably be expected to be highly correlated with those concepts. Therefore, in order to get (Q/h) to increase when we have $qup_t$ (which contains only positive values of $dq_t$ by construction), the sign of the coefficient on $qup_t$ must be positive. Moreover, to get (Q/h) to decrease when we have $qdown_t$ (which consists only of negative values of $dq_t$ by construction), the sign of the coefficient on $qdown_t$ must also be positive. Note that the absolute value of the $qup_t$ or $qdown_t$ coefficients should be less than one. Otherwise, this would indicate that employment would decrease in an upturn.

The $qpeak_t$ variable attempts to measure capacity output, both in the sense of capital and "hoarded" labour. In some cases, procyclical effects were not found to be very pronounced and variable $dq_t$ achieved better fits to the data. In this case, the sign on $dq_t$ should be positive, and less than 1. The key difference between this and the $qup_t / qdown_t$ formulation is that the $dq_t$ formulation constrains the upward and downward output elasticities to be the same. Finally, occasionally separating out effects associated with growth in real output and nominal affects associated with price changes (as modelled by industry price variable $pqag_t$) achieved the best fit.

In estimating the labour productivity equations, two particular forms were adopted. First, a "static" version of the functional form was employed where only the contemporaneous values of $qup_t$, $qdown_t$, $dq_t$ and $pqag_t$ are utilised. As far as static specifications are concerned, four particular functional forms are employed in the model. These are for industry i and time t:

(a) $$\ln\left(\frac{Q_{i,t}}{h_{i,t}}\right) = \alpha_0 + \alpha_1 * time_t + \alpha_2 * qdown_{i,t} + \alpha_3 * qup_{i,t};$$

(b) $$\ln\left(\frac{Q_{i,t}}{h_{i,t}}\right) = \alpha_0 + \alpha_1 * time_t + \alpha_2 * qdown_{i,t} + \alpha_3 * qup_{i,t} + \alpha_4 * pqag_{i,t};$$

(c) $$\ln\left(\frac{Q_{i,t}}{h_{i,t}}\right) = \alpha_0 + \alpha_1 * time_t + \alpha_2 * dq_{i,t};$$ and

(d) $$\ln\left(\frac{Q_{i,t}}{h_{i,t}}\right) = \alpha_0 + \alpha_1 * time_t + \alpha_2 * dq_{i,t} + \alpha_3 * pqag_{i,t}.$$

The second version is a "dynamic" version employing distributed lags of $qup_t$, $qdown_t$, $dq_t$ and $pqag_t$ with the sum of coefficients in the distributed lag structures being constrained to be less than one in absolute value. The structure of the dynamic equations employed in the model is listed in Table 16. In this table, the numbers such as "0-2" under a regressor variable indicates that a distributed lag structure involving 2 lags and the contemporaneous value of the regressor variable is employed.

**Table 16. Distributed Lag Structure of Dynamic Labour Productivity Regressions**

| Type | Intercept | Time Trend | qdown | qup | dq | pqag |
|---|---|---|---|---|---|---|
| e | + | + | 0-2 | 0-2 | | |
| f | + | + | 0-1 | 0-1 | | 0-1 |
| g | + | + | | | 0-2 | |
| h | + | + | | | 0-2 | 0-1 |

Table 17 lists the soft constraint specifications employed in the static labour productivity equations. No soft constraints were imposed on specifications involving variable $dq_t$ (equation types c and d). In table 17, for each regressor variable ($qdown_t$, $qup_t$, $pqag_t$), three columns are displayed – "target" defines the target value employed in the g7 "con" command; "actual" contains the actual result obtained using the soft constraint; and "unconst" denotes the value obtained from the unconstrained regression.

For variables $qdown_t$ and $qup_t$, soft constraints were used to either obtain a positive value when the unconstrained coefficient was negative or to reduce the magnitude of a positive coefficient so that its value was below one or to possibly bump up its explanatory power as measured by the "mexval" measure. For variable $pqag_t$, constraints were typically used to ensure that the value of the coefficient was less than one in absolute value or, once again, in order to bump up its explanatory power.

**Table 17. Soft Constraint Details for Estimation of Static Labour Productivity Equations**

| Industry | Type | $qdown_t$ | | | $qup_t$ | | | $pqag_t$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | target | actual | unconst | target | actual | unconst | target | actual | unconst |
| 1 | a | | | | | | | | | |
| 2 | d | | | | | | | | | |
| 3 | b | 0.9 | 0.954 | 1.229 | | | | -0.9 | -0.939 | -1.000 |
| 4 | b | 0.1 | 0.095 | -2.429 | 0.1 | 0.085 | 0.028 | | | |
| 5 | c | | | | | | | | | |
| 6 | b | 0.4 | 0.378 | -0.009 | | | | -0.2 | -0.158 | -0.059 |
| 7 | b | | | | | | | | | |
| 8 | a | 0.2 | 0.186 | 0.117 | | | | | | |
| 9 | a | | | | | | | | | |
| 10 | b | 0.1 | 0.045 | -0.453 | | | | | | |
| 11 | c | | | | | | | | | |
| 12 | b | 0.1 | 0.097 | -34.015 | 0.1 | 0.086 | 0.195 | | | |
| 13 | b | 0.2 | 0.164 | -0.053 | | | | | | |
| 14 | b | | | | | | | | | |
| 15 | b | 0.2 | 0.198 | -0.219 | | | | | | |
| 16 | a | 0.1 | 0.096 | -0.347 | | | | | | |
| 17 | b | 0.1 | 0.086 | -1.689 | 0.9 | 0.989 | 1.958 | | | |
| 18 | a | 0.98 | 0.980 | 2.577 | 0.1 | 0.075 | -0.916 | | | |
| 19 | a | 0.1 | 0.099 | -1.497 | | | | | | |
| 20 | b | 0.1 | 0.095 | -1.855 | 0.1 | 0.090 | 0.061 | -0.9 | -0.961 | -1.280 |
| 21 | b | 0.1 | 0.090 | -2.966 | 0.1 | 0.058 | 0.016 | -0.95 | -0.865 | -0.390 |
| 22 | c | | | | | | | | | |
| 23 | b | 0.95 | 0.950 | 10.409 | 0.1 | 0.109 | 0.170 | -0.95 | -0.946 | -0.928 |
| 24 | b | 0.1 | 0.096 | -0.901 | | | | | | |

In Table 18, the results from the softly constrained static regressions are listed. All coefficients have the correct signs and plausible magnitudes. It is evident that 18 of 24 equations have adjusted $R^2$'s above 0.6 while 11 of 24 equations have $R^2$'s above 0.9. The imposition of the soft constraints collectively do not significantly reduce the goodness of fit of the equations as measured by the adjusted $R^2$'s. Specifically, 10 equations experience a marginal reduction in the value of adjusted $R^2$ (industries 3, 4, 6, 10, 12, 13, 15, 17, 23, and 24 – compare columns 8 and 12 in Table 18). Four equations experience a more significant decline – these being the equations associated with industries 18, 19, 20 and 21. Even in these latter cases, however, the size of the decline is not significant in magnitude.

**Table 18. Estimation results for Static labour Productivity Equations**

| Industry | Intcp | Time | $qdown_t$ | $qup_t$ | $dq_t$ | $pqag_t$ | RSQR | MAPE | SEE | RHO | RSQRuc |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.181 | 0.027 | 0.929 | 0.390 | | | 0.944 | 1.59 | 0.05 | 0.47 | 0.944 |
| 2 | 3.977 | 0.075 | | | 0.281 | -0.549 | 0.981 | 1.09 | 0.07 | 0.19 | 0.981 |
| 3 | 3.831 | 0.053 | 0.954 | 0.192 | | -0.939 | 0.896 | 1.95 | 0.09 | 0.36 | 0.906 |
| 4 | 3.083 | 0.023 | 0.095 | 0.085 | | -0.101 | 0.918 | 1.02 | 0.04 | 0.60 | 0.922 |
| 5 | 2.585 | 0.012 | | | 0.794 | | 0.642 | 2.18 | 0.07 | 0.68 | 0.642 |
| 6 | 3.116 | 0.017 | 0.378 | 0.661 | | -0.158 | 0.860 | 0.84 | 0.04 | 0.55 | 0.868 |
| 7 | 3.067 | 0.033 | 0.144 | 0.768 | | -0.243 | 0.927 | 1.18 | 0.05 | 0.60 | 0.927 |
| 8 | 2.941 | 0.020 | 0.186 | 0.753 | | | 0.894 | 1.29 | 0.05 | 0.56 | 0.894 |
| 9 | 2.778 | 0.024 | 0.251 | 0.516 | | | 0.942 | 1.01 | 0.04 | 0.72 | 0.942 |
| 10 | 2.590 | 0.030 | 0.045 | 0.361 | | -0.108 | 0.975 | 0.76 | 0.03 | 0.26 | 0.979 |
| 11 | 2.676 | 0.002 | | | 0.491 | | 0.092 | 2.34 | 0.08 | 0.67 | 0.092 |
| 12 | 3.089 | 0.076 | 0.097 | 0.086 | | -0.665 | 0.966 | 1.71 | 0.08 | 0.73 | 0.969 |
| 13 | 3.056 | 0.015 | 0.164 | 0.325 | | -0.195 | 0.393 | 1.61 | 0.07 | 0.85 | 0.398 |
| 14 | 2.677 | 0.034 | 0.131 | 0.261 | | -0.669 | 0.930 | 0.72 | 0.03 | 0.48 | 0.930 |
| 15 | 2.997 | 0.010 | 0.198 | 0.495 | | -0.456 | 0.748 | 0.91 | 0.03 | 0.50 | 0.749 |
| 16 | 2.803 | 0.026 | 0.096 | 0.638 | | | 0.981 | 0.66 | 0.03 | 0.63 | 0.981 |
| 17 | 2.124 | 0.067 | 0.086 | 0.989 | | -0.103 | 0.983 | 1.44 | 0.06 | 0.51 | 0.986 |
| 18 | 3.364 | 0.020 | 0.980 | 0.075 | | | 0.721 | 1.94 | 0.09 | 0.93 | 0.766 |
| 19 | 3.626 | -0.004 | 0.099 | 0.352 | | | 0.370 | 0.72 | 0.03 | 0.46 | 0.395 |
| 20 | 3.432 | 0.037 | 0.095 | 0.090 | | -0.961 | 0.727 | 0.99 | 0.04 | 0.54 | 0.749 |
| 21 | 3.076 | 0.035 | 0.090 | 0.058 | | -0.865 | 0.342 | 1.18 | 0.04 | 0.67 | 0.459 |
| 22 | 2.870 | 0.012 | | | 0.306 | | 0.953 | 0.53 | 0.02 | 0.25 | 0.953 |
| 23 | 3.451 | 0.031 | 0.950 | 0.109 | | -0.946 | 0.524 | 0.92 | 0.04 | 0.29 | 0.528 |
| 24 | 3.133 | 0.014 | 0.096 | 0.244 | | -0.487 | 0.509 | 0.82 | 0.03 | 0.64 | 0.518 |

Table 19 lists the soft constraint specifications employed in the dynamic labour productivity equations. In table 19, for each autoregressive distributed lag specification associated with each broad regressor variable (relating to $qdown_t$, $qup_t$, $dq_t$, $pqag_t$), three columns are displayed – "target" defines the target value employed (in terms of the summation value of ADL process) in the g7 "con" command; "actual" contains the actual (summation) result obtained using the soft constraint; and "unconst" denotes the summation value obtained from the ADL structure in the unconstrained regression. For variables $qdown_t$, $qup_t$ and $dq_t$, soft constraints were used to either obtain a positive value when the sum of the unconstrained coefficients in the ADL were negative or to reduce the magnitude of a positive sum of ADL coefficient so that its value was below one or to possibly bump up its explanatory power as measured by the "mexval" measure.

For variable $pqag_t$, constraints were typically used to ensure that the value of the sum of the ADL coefficients were less than one in absolute value or, once again, to bump up its explanatory power. Finally, it should be noted that an "**na**" in the "target" columns indicates that no constraints were imposed on that respective regressor variable using the g7 "con" command. It is still possible, however, that the actual values will differ from the unconstrained values listed in the table because of the use of either the g7 "sma" command or through the effect of other "con" commands operating on the regressors that are not explicitly softly constrained.

**Table 19. Soft Constraint Details for Estimation of Dynamic Labour Productivity Equations involving $qdown_t$, $qup_t$ and $pqag_t$**

| Industry | Type | | $\sum qdown_t$ | | | $\sum qup_t$ | | | $\sum pqag_t$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | sma | target | actual | unconst | target | actual | unconst | target | actual | unconst |
| 1 | e | *(1) | 0.98 | 0.98 | 1.17 | 0.98 | 0.98 | 1.66 | | | |
| 3 | f | *(1) | 0.96 | 0.97 | 1.76 | **na** | 0.46 | 0.22 | -0.9 | -0.95 | -0.95 |
| 4 | f | *(1) | 0.5 | 0.50 | -1.09 | 0.3 | 0.30 | 0.22 | -0.2 | -0.20 | -0.18 |
| 6 | f | *(1) | 0.4 | 0.39 | 0.60 | **na** | 0.77 | 0.47 | **na** | -0.08 | -0.14 |
| 7 | f | *(1) | 0.5 | 0.50 | 0.01 | 0.5 | 0.49 | -0.05 | **na** | -0.34 | -0.50 |
| 8 | e | *(1) | 0.3 | 0.30 | -0.11 | 0.9 | 0.90 | 1.72 | | | |
| 9 | e | *(1) | 0.5 | 0.49 | 0.70 | 0.5 | 0.48 | -0.14 | | | |
| 10 | f | *(1) | 0.5 | 0.41 | -0.60 | 0.5 | 0.48 | 0.81 | **na** | -0.10 | -0.23 |
| 12 | f | *(1) | 0.5 | 0.50 | -63.89 | 0.5 | 0.47 | -2.13 | **na** | -0.66 | -0.52 |
| 13 | f | *(1) | 0.3 | 0.24 | -0.94 | **na** | 0.24 | 0.71 | **na** | -0.24 | -0.36 |
| 14 | f | *(1) | **na** | 0.32 | 0.25 | **na** | 0.46 | 0.65 | **na** | -0.70 | -0.73 |
| 15 | f | *(1) | 0.98 | 0.98 | 1.90 | **na** | 0.73 | 0.78 | **na** | -0.44 | -0.40 |
| 16 | e | *(1) | 0.5 | 0.50 | -2.06 | 0.98 | 0.98 | 1.22 | | | |
| 17 | f | *(1) | 0.5 | 0.50 | -2.09 | 0.98 | 0.98 | 2.54 | **na** | -0.07 | -0.10 |
| 18 | e | *(1) | 0.98 | 0.98 | 5.74 | 0.5 | 0.47 | -1.36 | | | |
| 19 | e | *(1) | 0.3 | 0.30 | -12.44 | 0.98 | 0.97 | 1.75 | | | |
| 20 | f | *(1) | 0.5 | 0.50 | -3.00 | 0.3 | 0.32 | 0.87 | -0.98 | -0.99 | -1.32 |
| 21 | f | *(1) | 0.3 | 0.30 | -6.44 | 0.3 | 0.27 | 0.93 | -0.9 | -0.84 | -0.33 |
| 23 | f | *(1) | 0.98 | 0.98 | 58.08 | 0.3 | 0.29 | -0.17 | -0.95 | -0.95 | -0.89 |
| 24 | f | *(1) | 0.3 | 0.30 | -0.91 | **na** | 0.38 | 0.45 | **na** | -0.43 | -0.44 |

**Table 19 (Cont). Soft Constraint Details for Estimation of Dynamic Labour Productivity Equations involving $dq_t$ and $pqag_t$**

| Industry | Type | | $\sum dq_t$ | | | $\sum pqag_t$ | | |
|---|---|---|---|---|---|---|---|---|
| | | sma | target | actual | unconst | target | actual | unconst |
| 2 | h | *(1) | 0.95 | 0.95 | 0.81 | **na** | -0.49 | -0.49 |
| 5 | g | *(1) | 0.98 | 0.98 | 1.42 | | | |
| 11 | g | *(1) | 0.5 | 0.49 | 0.24 | | | |
| 22 | g | *(1) | **na** | 0.58 | 0.70 | | | |

The estimation results associated with the dynamic labour productivity equations are listed in Table 20. It should be noted that the summations of the autoregressive distributed lags (ADL) structures of the regressor variables all have correct signs and plausible magnitudes. It is also evident that 17 of 24 equations have adjusted $R^2$'s above 0.6 while 11 of 24 equations have $R^2$'s above 0.9. As with the case of the static regressions, the imposition of the soft constraints collectively do not significantly reduce the goodness of fit of the equations as measured by the adjusted $R^2$'s.

**Table 20. Estimation Results for Dynamic Labour Productivity Equations involving $qdown_t$, $qup_t$ and $pqag_t$**

| Industry | Intcp | Time | $\sum qdown_t$ | $\sum qup_t$ | $\sum pqag_t$ | RSQR | MAPE | SEE | RHO | RSQRuc |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.153 | 0.027 | 0.98 | 0.98 | | 0.942 | 1.48 | 0.05 | 0.10 | 0.959 |
| 3 | 3.804 | 0.053 | 0.97 | 0.46 | -0.95 | 0.893 | 1.76 | 0.09 | 0.38 | 0.951 |
| 4 | 3.088 | 0.026 | 0.50 | 0.30 | -0.20 | 0.903 | 1.04 | 0.04 | 0.62 | 0.913 |
| 6 | 3.108 | 0.015 | 0.39 | 0.77 | -0.08 | 0.836 | 0.83 | 0.04 | 0.59 | 0.863 |
| 7 | 3.073 | 0.036 | 0.50 | 0.49 | -0.34 | 0.912 | 1.18 | 0.05 | 0.49 | 0.929 |
| 8 | 2.935 | 0.020 | 0.30 | 0.90 | | 0.875 | 1.29 | 0.05 | 0.62 | 0.886 |
| 9 | 2.785 | 0.024 | 0.49 | 0.48 | | 0.924 | 1.07 | 0.05 | 0.69 | 0.934 |
| 10 | 2.596 | 0.029 | 0.41 | 0.48 | -0.10 | 0.964 | 0.86 | 0.03 | 0.36 | 0.976 |
| 12 | 3.050 | 0.077 | 0.50 | 0.47 | -0.66 | 0.958 | 1.77 | 0.08 | 0.74 | 0.973 |
| 13 | 3.056 | 0.017 | 0.24 | 0.24 | -0.24 | 0.284 | 1.63 | 0.07 | 0.85 | 0.405 |
| 14 | 2.664 | 0.035 | 0.32 | 0.46 | -0.70 | 0.936 | 0.64 | 0.02 | 0.47 | 0.947 |
| 15 | 2.994 | 0.009 | 0.98 | 0.73 | -0.44 | 0.733 | 0.90 | 0.03 | 0.52 | 0.757 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 2.789 | 0.026 | 0.50 | 0.98 | | 0.979 | 0.61 | 0.02 | 0.66 | 0.983 |
| 17 | 2.111 | 0.066 | 0.50 | 0.98 | -0.07 | 0.979 | 1.43 | 0.06 | 0.51 | 0.984 |
| 18 | 3.354 | 0.019 | 0.98 | 0.47 | | 0.622 | 2.12 | 0.09 | 0.93 | 0.734 |
| 19 | 3.614 | -0.005 | 0.30 | 0.97 | | 0.303 | 0.68 | 0.03 | 0.45 | 0.753 |
| 20 | 3.412 | 0.039 | 0.50 | 0.32 | -0.99 | 0.704 | 0.96 | 0.04 | 0.53 | 0.740 |
| 21 | 3.049 | 0.035 | 0.30 | 0.27 | -0.84 | 0.280 | 1.13 | 0.04 | 0.71 | 0.424 |
| 23 | 3.434 | 0.031 | 0.98 | 0.29 | -0.95 | 0.414 | 0.93 | 0.04 | 0.33 | 0.515 |
| 24 | 3.131 | 0.012 | 0.30 | 0.38 | -0.43 | 0.382 | 0.82 | 0.03 | 0.63 | 0.527 |

**Table 20 (Cont). Estimation Results for Dynamic Labour Productivity Equations involving $dq_t$ and $pqag_t$**

| Industry | Inctp | Time | $\sum dq_t$ | $\sum pqag_t$ | RSQR | MAPE | SEE | RHO | RSQRuc |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 3.907 | 0.074 | 0.95 | -0.49 | 0.983 | 0.98 | 0.06 | 0.17 | 0.985 |
| 5 | 2.583 | 0.012 | 0.98 | | 0.632 | 2.11 | 0.07 | 0.67 | 0.663 |
| 11 | 2.674 | 0.002 | 0.49 | | 0.011 | 2.31 | 0.08 | 0.71 | 0.073 |
| 22 | 2.856 | 0.012 | 0.58 | | 0.950 | 0.53 | 0.02 | 0.25 | 0.951 |

**(2) Average Hours Worked Equations**

The equations for average hours worked relate annual hours worked per employee (annual average hours worked) to time trends and changes in output, much like the labor productivity equations. “Static” versions of the equations are currently available. These are based on first differenced values of industry output and an assortment of time trends that match the particular characteristics of industry average hours worked time series data. The coefficients associated with the first difference of industry output are constrained (where appropriate) to be a positive value.

A number of different time trends had to be used to obtain acceptable results. Specifically, the following time trends and dummy variables were defined:

- t1 = a linear time trend encompassing the whole sample period 1970 to 2001;
- t2 = a time trend starting in 1985;
- t3 = a time trend starting in 1989;
- t4 = a time trend starting in 1986;
- t5 = a time trend starting in 1984; and
- dum2 = impulse dummy variable taking a value of one in 1986 and zero elsewhere.

The choice of the starting dates was chosen solely on the basis of apparent changes in trends observed in time plots of the dependent variable time series for each industry. As such, these time trends explain much more than the first differenced output terms and, because of this, these equations can be viewed as essentially modelling observed time trends in the dependent variables. The equations contain minimal economic content capable of explaining the observed trends. This topic will be the subject of ongoing research as the model develops. The results for the static equations are listed in Table 21.

**Table 21. Estimation Results for Static Average Hours Worked Equations**

| Industry | Intcp | time | dq | t2 | t3 | t4 | t5 | dum2 | RSQR | MAPE | SEE | RHO | RSQRuc |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2641.22 | -18.53 | 46.75 | | | | | | 0.882 | 1.89 | 50.3 | 0.56 | 0.882 |
| 2 | 1886.21 | -11.57 | 51.99 | 35.61 | | | | | 0.922 | 1.45 | 34.2 | 0.16 | 0.928 |
| 3 | 2183.56 | -11.88 | 44.66 | | | 34.80 | | | 0.853 | 1.60 | 41.4 | 0.17 | 0.885 |
| 4 | 1987.04 | -12.58 | 150.29 | | | | 17.94 | | 0.557 | 1.09 | 24.8 | -0.10 | 0.557 |
| 5 | 1983.37 | -11.04 | 46.76 | | | | 12.94 | | 0.296 | 1.12 | 27.1 | 0.30 | 0.306 |
| 6 | 1916.56 | -9.20 | 71.64 | | | | 14.12 | | 0.407 | 1.14 | 26.8 | 0.36 | 0.410 |
| 7 | 2048.60 | -12.31 | 49.94 | | | | 19.40 | | 0.656 | 0.94 | 24.0 | 0.21 | 0.656 |
| 8 | 2071.68 | -10.11 | 60.54 | | | | 17.09 | | 0.498 | 1.32 | 32.8 | 0.37 | 0.534 |
| 9 | 2058.35 | -12.45 | 63.17 | | | | 20.53 | | 0.628 | 1.16 | 29.2 | 0.45 | 0.710 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 2070.06 | -14.53 | 60.55 | | | | 23.10 | | 0.728 | 0.92 | 25.3 | 0.29 | 0.786 |
| 11 | 1994.71 | -14.41 | 197.36 | | | | 23.68 | | 0.239 | 2.71 | 69.1 | -0.22 | 0.240 |
| 12 | 1974.20 | -18.68 | 34.93 | 31.92 | | | | | 0.810 | 1.06 | 28.3 | 0.15 | 0.810 |
| 13 | 2029.41 | -13.63 | 282.58 | 20.81 | | | | | 0.742 | 0.96 | 21.8 | 0.40 | 0.742 |
| 14 | 1910.50 | -8.22 | 97.85 | | | | | | 0.924 | 0.82 | 17.4 | 0.44 | 0.924 |
| 15 | 1772.10 | -5.37 | 168.00 | | | | | | 0.666 | 1.32 | 27.3 | 0.47 | 0.666 |
| 16 | 2050.32 | -7.24 | 147.72 | | 15.96 | | | | 0.646 | 0.87 | 21.5 | 0.66 | 0.656 |
| 17 | 1828.82 | -13.90 | 34.69 | 29.72 | | | | | 0.896 | 1.18 | 24.5 | 0.32 | 0.896 |
| 18 | 1736.18 | -1.66 | 56.36 | 10.02 | | | | | 0.727 | 1.27 | 26.4 | 0.65 | 0.755 |
| 19 | 1879.50 | -5.69 | 100.01 | 9.38 | | | | | 0.469 | 0.77 | 16.9 | 0.53 | 0.469 |
| 20 | 1771.93 | -5.81 | 194.94 | 8.84 | | | | | 0.608 | 0.53 | 11.8 | 0.15 | 0.608 |
| 21 | 1651.72 | 1.67 | 200.12 | | | | | -144.32 | 0.603 | 1.03 | 22.0 | 0.38 | 0.609 |
| 22 | 1692.32 | -5.71 | 145.28 | | | | | | 0.921 | 0.67 | 12.5 | 0.16 | 0.922 |
| 23 | 1577.27 | 4.47 | 20.72 | | -11.99 | | | | 0.569 | 0.95 | 20.6 | 0.44 | 0.580 |
| 24 | 1532.56 | 4.45 | 422.50 | | | | | | 0.808 | 0.89 | 17.6 | 0.29 | 0.808 |

Table 21 shows that there has been an overall trend decline in averaged hours worked across all industries, except possibly the last, as captured by the coefficients associated with the variable “time” in the table above. In the case of industry 21, however, the large negative value of the impulse dummy variable could be masking an apparent slight overall trend decline in average hours worked in this industry. The positive signs associated with the coefficients associated with the trends t2, t3, t4 and t5 denote an apparent shift in this observed trend decline from the mid 1980’s, however, to an observed trend increase over the remainder of the sample period. Therefore, many industries seemed to have experienced a strong trend decline in average hours worked from 1975 to the mid 1980’s, followed by an apparent reversal over the remainder of the sample period. Finally, it should be noted that the imposition of soft constraints on the first differenced output variable (dq) did not produce a marked deterioration in the goodness of fit of the equations as measured by the adjusted $R^2$. This can be discerned by examining the values for the softly constrained regressions (see column 10 of Table 21 titled “RSQR”) with the values for the unconstrained regression (see column 14 of Table 21 titled “RSQRuc”).

**(3) Employed Persons definition of Employment**

The employed persons measure of total employment by industry is obtained by dividing total hours worked (whether determined from the GL model or the labour productivity equations) by average hours worked per employee. This yields the civilian based employed person measures for all industries (both private and public) that are published by the ABS in its labour market survey publications. Total employment is then obtained by adding military employment to the civilian based employment measures. Military employment is specified exogenously.

**LABOUR MARKET MODULE: MACROECONOMIC DETAILS[9]**

**Key Endogenous Variables:**

. long run supply of aggregate average hours worked
. labour supply equation
. beveridge curve relation and skills adjusted unemployment rate
. aggregate employment
. unemployment rate
. labour participation rate
. aggregate labour productivity
. Aggregate Wage Setting relation and NAIRU

**Discussion**

[9] Key references for this section are Commonwealth Treasury (1996) and Thomson (2000).

The labour market module is designed to balance decisions concerning the demand for and supply of labour. The labour supply equation is specified as a function of the level of employment on an hours worked basis, reflecting the discouraged/encouraged worker effect. Real wages, a variable time trend (to capture the upward trend in the participation rate) and demographic dummy variables (to capture the effects of baby boomers, increased lifespan and increased female participation) are also used in the labour supply equation. The dependent variable in this equation is the participation rate adjusted for hours worked (in order to accurately reflect the long-run desired level of hours worked).

The Australian Bureau of Statistics (ABS) average hours worked series reflects both demand and supply influences. To ensure that the labour supply equation is identified, demand influences are removed by fitting a logistic growth function to the published hours worked series. This reflects the decline in average hours worked since the early 1970s. The derived series serves as a proxy for the long-run desired level of hours worked.

A Beveridge curve equation (describing the relationship between the unemployment rate and the vacancy rate) is introduced to endogenise the unfilled job vacancy series and to provide a measure of the state of the labour market for the aggregate wage equation. The equation utilises a logistic growth function to capture the outward movement of the Beveridge curve relationship that occurred in the 1970s.

Aggregate wage setting behaviour is explained using an expectations augmented Phillips curve. In the long-run, wages move in line with underlying labour productivity. Other factors such as expected consumer price inflation, the degree of excess demand in the labour market and changes in the unemployment rate (adjusted to reflect the influence of insiders on wage behaviour) also have an effect in the short-run.

Labour demand is determined at the industry level, and then aggregated to give total labour demand. Recall that the modeller may utilise either of two estimation methods to determine industry labour demand on a total hours worked basis. First, as mentioned previously, equations are estimated for industry labour productivity and industry average annual hours worked per employee. Industry total hours worked is then obtained by dividing industry output by estimated industry labour productivity. Second, industry capital and labour demand (total hours worked) can be estimated simultaneously using the Generalised Leontief cost function and combined with the estimated equations for industry average annual hours worked per employee mentioned above. Industry labour demand (on an employed persons basis) is then calculated by identity by dividing industry total hours worked by industry average hours worked per employee.

**Long run supply of aggregate average hours**

In the long-run, aggregate average hours worked moves to a supply equilibrium estimated using a logistic growth function. This gives the long-run equilibrium level of hours worked (NHLR), which reflects a downward trend over time due to increased part-time employment and a shortening of the standard work week in the mid-1970s. The functional form adopted is:

$$\log(nh_t) = \underset{(+)}{\beta_0} + \underset{(-)}{\beta_1} / \left(1 + \exp(\underset{(-)}{\beta_2} * (qtime + \underset{(+)}{\beta_3}))\right)$$

where:

- $nh_t$ is an index variable with a value of one in 1996/97 that is constructed from the ratio of total hours worked to total employed persons; and
- $qtime_t$ is a linear time trend.

The logistic function is an S-shaped curve that acts as a structural break dummy variable, allowing the data to determine the size and timing of any break in the series. The timing of the break is estimated by the $\beta_3$ parameter, while the $\beta_1$ and $\beta_2$ parameters reflect the size and slope of the shift respectively.

The following estimates for the long-run coefficients were obtained from the non-linear estimation technique based on the simplex method contained in the g7 regression package:

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\beta_0$ | 0.141 | 6.13 |
| $\beta_1$ | -0.142 | 5.97 |
| $\beta_2$ | -0.338 | 5.58 |
| $\beta_3$ | 26.125 | 23.15 |

**Labour supply equation**

The dependent variable for the labour supply equation is given by the ratio of the labour force (LFC) to the population aged 15 to 64 years (NPADAA), ie. the participation rate. This is adjusted for long-run desired hours worked (NHLR). In the long-run, labour supply is also a function of the employment rate adjusted for NHLR and increased female participation, a variable time trend which captures upward movement in the participation rate over time due to changing preferences, and demographic effects. This gives the following long-run equilibrium equation for labour supply:

$$\log\left(\frac{lfc_t * nhlr_t}{npadaa_t}\right) = \underset{(+)}{\alpha_0} * qnlfal_t * \log\left(\frac{totemp_t * nhlr_t}{npadaa_t}\right) + trend_t + \log(qdeml_t)$$

where:

- $lfc_t$ is the labour force;
- $nhlr_t$ is the long-run desired hours worked;
- $npadaa_t$ is the population aged 15 to 64 years;
- $totemp_t$ is total employment;
- $qdeml_t$ is an index variable with a value of one in 1996/97 that is constructed by applying participation rates by age cohort to population proportions over time;
- $qnlfal_t$ is an index variable with a value of one in 1996/97 that is constructed from the ratio of total number of females employed to the labour force; and
- $trend_t$ is a variable time trend, defined as follows:

$$trend_t = \begin{array}{l} \alpha_1 + \alpha_2 / \left(1+\exp(\alpha_3 * (qtime + \alpha_4))\right) \\ (+) \ (-) \qquad\qquad (-) \qquad\qquad (+) \\ + \alpha_5 * \{qtime - pos(qtime + \alpha_6) \\ \ (+) \qquad\qquad\qquad\qquad\qquad (+) \\ + \alpha_7 * [4/(1+\exp(-(pos(qtime+\alpha_6))/\alpha_7)) - 2]\} \\ \ (+) \qquad\qquad\qquad\qquad\qquad\qquad (+) \end{array}$$

The time trend variable captures variations in the rate of increase in the participation rate over time due to social factors such as greater female participation and education retention. The functional form takes into account the fact that the upward trend cannot lift the participation rate over 100 percent, while allowing the timing and size of shifts to be driven by the data.

There are three parts to the demographic effects. First, the effect of the baby boom moving through the age cohorts over time, leading to variations in the participation rate is measured by the variable QDEML. Second, increased lifespan has meant that the participation rate (typically measured on a 15 years and over basis) has tended to bias down the upward trend in the participation rate, since the proportion of the population aged 65 years and over is growing, but contains almost no participation. To adjust for this, the participation rate is estimated on a 15 to 64 years basis.  Third, the impact on labour force participation of the increase in female participation is modelled by the variable QNLFAL.

The following estimates for the long-run coefficients were obtained from the simplex based non-linear estimation technique in the g7 regression package:

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\alpha_0$ | 0.430 | 7.13 |
| $\alpha_1$ | 2.242 | 0.83 |
| $\alpha_2$ | -2.032 | -0.77 |
| $\alpha_3$ | -0.124 | -2.51 |
| $\alpha_4$ | 23.734 | 68.13 |
| $\alpha_5$ | 0.059 | 1.03 |
| $\alpha_6$ | 19.134 | 9.46 |
| $\alpha_7$ | 6.425 | 2.09 |

An error correction (ecm) specification is used to incorporate the dynamic and long-run responses:

$$\log\left(\frac{lfc_t * nhlr_t}{npadaa_t}\right) =$$

$$\beta_0 * d\log\left(\left(\frac{(1-labtax_{t-1}) * rtn_{t-1}}{pcon_{t-1}}\right) - lambda_{t-1}\right) + \beta_1 * d\log\left(\left(\frac{(1-labtax_{t-2}) * rtn_{t-2}}{pcon_{t-2}}\right) - lambda_{t-2}\right)$$

(+) (+)

$$+\beta_2 * \left(qnlfal_t * d\log\left(\frac{totemp_t * nhlr_t}{npadaa_t}\right)\right) + \beta_3 * \left(qnlfal_{t-1} * d\log\left(\frac{totemp_{t-1} * nhlr_{t-1}}{npadaa_{t-1}}\right)\right) + \beta_4 * dtrend$$

(+) (+) (+)

$$+\beta_5 * \left(\log\left(\frac{lfc_{t-1} * nhlr_{t-1}}{npadaa_{t-1}}\right) - \left(\overline{\alpha_0} * qnlfal_{t-1} * \log\left(\frac{totemp_{t-1} * nhlr_{t-1}}{npadaa_{t-1}}\right) + \log(qdeml_{t-1}) + (trend_{t-1})\right)\right)$$

(−)

where:

- $labtax_t$ is the tax rate on labour income;
- $rtn_t$ is nominal wages;
- $pcon_t$ is the household consumption price deflator; and
- $lambda_t$ is the annual rate of technical progress.

Because of the complexity of the above equation relative to the rather modest sample size available for estimation purposes, the Engle-Granger two-step procedure was utilized in order to estimate the ecm equation using a non-linear estimation procedure. Thus, the long run coefficients were imposed in the ecm equation after being obtained from the long run equation which was estimated separately. In the short-run, changes in the participation rate also depend on changes in lagged after-tax consumer real wages, and the contemporaneous and lagged employment rate (to capture the impact of the encouraged worker effect). It should be noted that in attempts to estimate the ecm equation, the sign of coefficient $\beta_0$ was found to be consistently negative which is not consistent with a priori expectations. However, an overall constraint was employed in the non-linear estimation procedure to ensure that the sum of coefficients $\beta_0$ and $\beta_1$ was positive, thus ensuring that $\beta_1$ is more positive than $\beta_0$ is negative. This meant that the overall effect of the after-tax consumer real wage on labour force participation was consistent with our a priori expectations.

The following estimates for the ecm coefficients were obtained from the simplex based non-linear estimation method in the g7 package:

| **Parameter** | **Coefficient Estimate** |
|---|---|
| $\beta_0$ | -0.009 |
| $\beta_1$ | 0.013 |
| $\beta_2$ | 0.357 |
| $\beta_3$ | 0.003 |
| $\beta_4$ | 0.920 |
| $\beta_5$ | -0.821 |

Note that the t statistics are not included because the imposition of soft constraints on the sum of coefficients $\beta_0$ and $\beta_1$ render their conventional interpretation invalid.

**Beveridge curve and skills adjusted unemployment rate**

The Beveridge curve equation is designed to describe the inverse relationship between unemployment and job vacancies. According to traditional theory, any reduction in the search effectiveness of the unemployed should be reflected in a rise in unfilled vacancies for a given level of unemployment and, therefore, an outward movement of the unemployment/vacancy relationship. Search effectiveness can be affected by a wide range of factors, including changes in the benefits system, the industrial structure, skill composition, or the responsiveness of wage relativities to changes in the pattern of demand.

The Beveridge curve equation is based on a dynamic error correction specification. A logistic growth function (LGF) is employed in the equation to capture the outward shift in the unemployment/vacancy relationship that appeared to have occurred in the mid-1970s.

The dynamic Beveridge curve equation is described below:

$$d\log(rnu_t) =$$

$$\underset{(-)}{\beta_0} * d\log\left(\frac{nva_t}{lfc_t}*100\right) + \underset{(+)}{\beta_1} * d(LGF) + \underset{(-)}{\beta_2} * \left(\log(rnu_{t-1}) - \left(LGF_{t-1} + \underset{(-)}{\overline{\alpha_4}} * \log\left(\frac{nva_{t-1}}{lfc_{t-1}}*100\right)\right)\right)$$

where:

- $rnu_t$ is the unemployment rate;
- $nva_t$ is the number of job vacancies; and
- $\mathrm{LGF} = \underset{(+)}{\alpha_0} + \underset{(+)}{\alpha_1} / \left(1 + \exp(-(qtime_t - \underset{(-)}{\alpha_2}) / \underset{(+)}{\alpha_3})\right)$

Once again, because of the complexity of the above equation relative to the modest sample available for estimation purposes, the Engle-Granger two-step procedure was utilized to estimate the ecm equation. As such, the long run coefficients $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$ were imposed in the ecm equation after being obtained from the long run equation, which was estimated separately. The coefficient estimates from both the long run $(\alpha's)$ and ecm $(\beta's)$ equations (utilising the simplex based non-linear estimation method in the g7 package) are:

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\alpha_0$ | 0.499 | 0.59 |
| $\alpha_1$ | 1.454 | 1.70 |
| $\alpha_2$ | -28.467 | -9.21 |
| $\alpha_3$ | 2.392 | 2.40 |
| $\alpha_4$ | -0.415 | -6.12 |
| $\beta_0$ | -0.423 | -6.95 |
| $\beta_1$ | 1.026 | 3.94 |

| | | |
|---|---|---|
| $\beta_2$ | -0.356 | -2.00 |

To enable identification of either search effectiveness or wage bargaining explanations for movements in the non-accelerating inflation rate of unemployment (NAIRU), the skills adjusted unemployment rate (RNUST) is introduced into the aggregate wage equation. This variable captures the impact on equilibrium unemployment of changes in search effectiveness. In order to calculate the variable RNUST, we equate the vacancy and unemployment rates and solve for the unemployment rate. Hence, the value for the unemployment rate will depend only on the LGF, as follows:

$$\text{RNUST} = \exp\left(1/(1+\alpha_4)*\left(\alpha_0 + \alpha_1/(1+\exp(-(qtime_t - \alpha_2)/\alpha_3))\right) + resid\_bc\right)$$

Note that $resid\_bc$ is the calculated residual from the Beveridge curve equation. This is added to rnust to ensure that unexplained movements in search effectiveness are incorporated in the nairu.

**Aggregate wage setting equation and NAIRU**

The modeller may utilise either of two estimation methods (TRYM- or AEM-based) to determine aggregate wage setting behaviour. Both methods are based on an expectations augmented Phillips curve specification and enable calculation of the nairu term.

In the TRYM-based equation,[10] wages are assumed to be dependent on changes in the household consumption price deflator (AVGINFL) and in labour productivity (AVGGRPRD) in the long-run. Central to this equation is the assumption that wage inflation is also dependent upon the degree of excess demand in the labour market. This is proxied by the differential between the actual rate of unemployment and the nairu. The nairu is estimated from the wage equation using historical data. A dummy variable (Q741) is included in the nairu calculation to account for an apparent upward shift around 1974/75.

Several theories attempt to explain the upward shift in the nairu. The labour market module does not contain the level of detail necessary to distinguish between them. However, incorporation of the skills adjusted unemployment rate (RNUST, derived from the Beveridge curve relationship) and two estimated wage-setting parameters (WS and WSo), allows us to distinguish between search effectiveness and wage bargaining factors affecting the nairu, which is derived as follows:

$$\text{NAIRU} = (\underset{(+)}{RNUST_t + WS})*(1-Q741_t) + (\underset{(+)}{RNUST_t + WSo})*Q741_t$$

In the short-run, wage inflation is dependent on changes in the unemployment rate. Because an increase in unemployment will generally have less effect on wages than an equivalent fall, the coefficient on the change in the unemployment rate is allowed to vary between positive and negative changes. In order to model the effects of those inside employment (insiders) on wage behaviour, changes in the unemployment rate are weighted by the proportion of employees that are union members (RUM). For example, outsiders may be viewed by employers as imperfect substitutes for insiders, due to factors such as regulations or transaction costs associated with hiring and firing. Therefore, because insiders' jobs are relatively more secure, their wage claim decisions are less sensitive to the unemployment rate.

[10] Consult Commonwealth Treasury (1996) and Thomson (2000, pp. 19-26).

Finally, a variable designed to capture the effects on wages of changes to the degree of centralisation of the wage determination system (QCC) has also been included in the TRYM-type specification. These features result in the following equation:

$$
\begin{aligned}
& d\log(wsspvt_t) = \\
& \underset{(+)}{\beta_0} * avggrprd_t + \underset{(+)}{\beta_1} * avg\inf l_t + \left(\underset{(-)}{\beta_2} * rum_t * (\underset{(+)}{\beta_3} * grunemp\_u_t + (1-\underset{(+)}{\beta_3}) * grunemp\_d_t)\right) \\
& + \underset{(-)}{\beta_4} * dqcc_t + \underset{(+)}{\beta_5} * \left(\left((rnust_{t-1} + \underset{(+)}{ws}) * (1 - q741_t) + (rnust_{t-1} + \underset{(+)}{wso}) * q741_t - rnu_t\right) / rnu_t\right)
\end{aligned}
$$

where:

- $wsspvt_t$ is an index (equal to one in 1996/7) of the average nominal wage rate per hour;
- $avggrprd_t$ is a weighted average of past changes in the percentage change in aggregate productivity, given by the ratio of GDP to total hours worked;
- $avg\inf l_t$ is a weighted average of past changes in the percentage change in the household final consumption expenditure price deflator;
- $rum_t$ is the rate of union membership;
- $grunemp\_u_t$ are lagged positive changes in the unemployment rate;
- $grunemp\_d_t$ are lagged negative changes in the unemployment rate; and
- $dqcc_t$ is the change in an index that acts as a proxy for the varying degree of centralisation in the wage determination system.

The coefficient estimates for the TRYM-based aggregate wage equation (determined using the simplex based non-linear estimation method in the g7 package) are as follows:

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\beta_0$ | 1.399 | 2.33 |
| $\beta_1$ | 0.747 | 4.61 |
| $\beta_2$ | -0.058 | -2.02 |
| $\beta_3$ | 0.412 | 1.46 |
| $\beta_4$ | -2.165 | -1.27 |
| $\beta_5$ | 7.460 | 3.51 |
| $ws$ | 3.049 | 1.78 |
| $wso$ | 1.851 | 1.67 |

In the AEM-based equation,[11] wages are also dependent on price inflation and labour productivity in the long-run. In the short-term, however, wages depend inversely on the unemployment rate in the previous year, on the lagged change in the unemployment rate, and on a supply price variable. This variable serves as a proxy for the effects of supply price shocks on wage inflation. These factors result in the AEM-based specification:

[11] Consult Thomson (2000, pp. 45-46) and Wild (1995, pp. 86-87, 426).

$d\log(wsspvt_t)$ =

$\beta_0 + \beta_1 * avggrprd_t + \beta_2 * avg\inf l_t + \beta_3 * rlab\_aem_t + \beta_4 * run\_aem_t + \beta_5 * \sup ply_t$

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where:

. $rlab\_aem_t$ is the lagged change in the unemployment rate;

. $run\_aem_t$ is the inverse of the unemployment rate in the previous year; and

. $\sup ply_t$ is a supply price variable derived by averaging the differentials between the GDP price deflator and the agriculture and petroleum products output price deflators.

The coefficient estimates for the AEM-based aggregate wage equation (determined using the conventional OLS command in the g7 package) are as follows:

| **Parameter** | **Coefficient Estimate** | **t-values** |
|---|---|---|
| $\beta_0$ | -4.649 | -2.24 |
| $\beta_1$ | 1.196 | 1.83 |
| $\beta_2$ | 0.736 | 3.92 |
| $\beta_3$ | 64.354 | 1.53 |
| $\beta_4$ | 28.296 | 1.41 |
| $\beta_5$ | 0.204 | 1.56 |

$R^2$ = 0.73, MAPE = 26.42, SEE = 1.84, RHO = 0.13.

In long-run equilibrium, the unemployment rate is constant and expected real wage inflation is equal to trend. Therefore, the nairu is equal to:

$$\text{NAIRU} = -\left(\frac{\beta_4}{\beta_0}\right) = -\left(\frac{28.296}{-4.649}\right) = 6.086.$$

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