The Craft of Economic Modeling

Part 2

Projected Contents

QUEST - A Quarterly Econometric Structural Model

1. Overview

In Part 1, we developed a very simple model and suggested some directions in which it could be expanded. In the present chapter, we will carry out some of the suggestions while trying to follow the good advice of the last chapter of Part 1. In particular, our model will

  refine the consumption and employment functions presented previously.

  divide fixed investment into three major components, equipment, residences, and other structures, and develop appropriate equations for each.

  develop equations for exports and imports.

  complete the income side of the model with equations for capital consumption, profits, dividends, interest rates, interest payments and income, employee compensation and proprietor income.

  calculate revenues from various taxes, government expenditures in current prices (from variables exogenous in constant prices), interest payments, and budgetary deficits or surpluses for the federal government and, separately, for the combination of state and local governments.

The word "structural" in the name of the Quest model is noteworthy. Quest is a model intended to embody and test an understanding of how the economy works. It is concerned with how aggregate demand affects employment, how employment affects unemployment, how unemployment affects prices, how prices and money supply affect interest rates and incomes, and how incomes, interest rates, and prices affect investment, consumption, imports, and exports, which make up aggregate. demand. The model embodies a view of how each link in this closed-loop chain works. Satisfactory performance is not to judged by how well it works forecasting a few quarters ahead, but by how well it holds up over a much longer period. Can it keep employment within a few percent of the labor force over decades? Can it keep inflation in line with the increase in money supply though it does not use money supply in the inflation equation? Can it right itself if thrown off course for a few quarters? We will test it in 21-year historical simulation, time enough for it to go seriously astray if it is inclined to do so.

In this respect, Quest is quite different from most quarterly models of my acquaintance. They are usually aimed at short-term forecasting, usually of not more than eight quarters. They can therefore make extensive use of lagged values of dependent variables in the regression equations. The use of these lagged dependent variables gives close fits but leaves little variability for identifying the parameters of the underlying structural equations, which are often rather weak in such models. Our interest centers in the structural equations. In estimating the equations of Quest, therefore, we

have avoided lagged values of dependent variables in the regression equations. When used for short-term forecasting, Quest uses the rho-adjustment method of error correction described in Chapter 2.

Models often have a special purpose, a special question they are designed to answer. Quest is basically a general-purpose marcoeconomic model, but it would be less than fair to the reader not to mention that there was a particular question on my mind as I worked on it in the summer of 1999. As in the summer of 1929, exactly seventy years earlier, the economy was growing strongly and the stock market was at unprecedented — and, quite possibly, unjustified — highs. The run-up in the stock market was generally attributed to the influx of footloose capital from Asian markets. At the first sign of a drop, this capital could leave as suddenly as it came. The stock market would then fall. But how would that fall affect employment and output in the real economy? As I revised the model in the summer of 2001, the stock market had declined significantly, and economic growth had slowed sharply. How far the fall would go and how sharp the recession would be was still unclear.

The stock market plays no role in the National Income and Product accounts, but its performance can make people feel wealthy or poor and thus influence how they spend or save. It determines how much equity in a firm must be diluted in order to raise a given amount of capital by issuing stock. In this way, it affects the cost of capital as perceived by the owners of companies, and thus may affect investment. We will enter the Standard & Poor index of the prices of 500 stocks as an explanatory variable in a number of behavioral equations, and finally we will try to explain this variable by corporate profits and interest rates. The variable proves very helpful in a number of the equations, but the attempt to explain it is only partly successful. In particular, the rise in 1997- 2000 is very incompletely explained. To test out the rest of the model, we run it with this equation turned off. To get an idea of where the economy would be after a "crash" back to levels explainable by profits and interest rates, we just run the model with it turned on. The results are, shall we say, "instructive." But first we must look at the equations.

2. The Behavioral Equations

Personal consumption expenditures

We work up to the main equation for personal consumption expenditures with two supporting equations, one for expenditures on motor vehicles and one for Interest paid by consumers to business. The interest paid variable is particularly relevant because consumers must pay it out of their disposable income but it is not part of personal consumption. Thus, if interest payments rise relative to disposable income, they must come out of either savings or consumption. We will find out which choice consumers make. The expenditures on motor vehicles is important for total expenditures for two reasons. First, interest payments on car loans is a major component of the Interest paid by consumers to business. (Interest on home mortgages is *not* part of Interest paid by consumers to business, because home ownership is considered a business in the NIPA.) Second, the NIPA consider that an automobile is consumed in the quarter in which it is purchased. Consumers, however, think of the car as being consumed over its lifetime. Thus, if automobile

purchases are particularly strong in a certain quarter, there is a sort of savings in the form of automobiles. It would not be surprising to see all or most of that saving appear as consumption in the NIPA series. Though the same reasoning applies to other durables, their purchases are much less volatile than those of automobiles, so there is not much to be gained by such treatment.

We start with **personal consumption expenditures on motor vehicles.** It uses real disposable income accrued per capita, yRpc, lagged values of its first difference, dyRpc, the Treasury bill rate, rtb, multiplied by yRpc as an indicator of credit conditions, and an estimate of the wear-out of motor vehicles, mvWear.

Disposable income accrued is in most quarters exactly the same as disposable income. In a few quarters, however, billions of dollars of bonuses that should normally have been paid in the fourth quarter of one year were, for tax reasons, paid in the first quarter of the next. Consumers definitely based their consumption on the accrued rather than the disbursed income. We will therefore almost always use Personal disposable income accrued, pidisa, not Personal disposable income, but we will call it simply "disposable income."

The increments in this real disposable income per capita are crucial variables in this equation. Their total is 1.28. Since we are dealing with quarterly flows at annual rates, this 1.28 implies that a rise in annual income of $1 leads to an increase in the stock of motor vehicles of $.32 (= 1.28×.25). We shall return below to look at the pattern of the coefficients.

The deviation of the interest rate, rtb, from a typical value, here taken as 5 percent, is multiplied by yRpc so that the amplitude of its swings will grow at approximately the same rate as the growth in the dependent variable.

The wear-out variable required more than the usual constant wear rate. When a constant rate was used, the equation under-predicted at the beginning of the period and over-predicted at the end. It is common experience that automobiles last longer now than they did thirty years ago, so a declining wear-out or "spill" rate, *spilla*, was introduced. It is 10 percent per quarter in 1974.4, just before the beginning of the fit period, and declines at 2 percent per year. The usual "unit bucket" way of correcting for initial filling of a bucket is not valid with a variable spill rate, but calculations showed that, at these spill rates, filling was not a problem after 15 years, the time between the beginning of the Quip bank and the beginning of this regression. Without any constraint, the coefficient on this variable came out at .99058, thus indicating almost exact dollar-for-dollar replacement of the cars wearing out.

On the other hand, the income variable, yRpc, was not used because, when included, it had a small negative coefficient. That does not mean that motor vehicle expenditures do not depend on income, but rather that the dependence comes about entirely by expansion of the stock in response to an increase in income and then replacement of that stock.

In this and subsequent presentations, we do not show the full "catch" file but only the part not obvious from the display of the regression results. Thus, the *catch, save, limits, r, gr, gname,* and *spr* commands have been deleted to save space.

```
ti Motor Vehicles
# cdmvRpc is per capita consumption of motor vehicles in constant dollars
fex cdmvRpc  = cdmvR/pop

#Disposable Income per Capita
fex pidisaR = pidisa/gdpD
f yRpc = pidisaR/pop
f dyRpc = yRpc - yRpc[1]

# Interest rate X ypcR to represent credit conditions
f rtbXypc = .01*(rtb -5.)*yRpc
# (Real rate was tried, but was much less effective.)

# Create wearout of automobiles assuming 8% per quarter wearout rate
f spilla = .10*@exp(-.02*(time -15.))
f mvWearpc = spilla*@cum(mvSt,cdmvR[1],spilla)/pop
sma 50000 a3 a11 1
:                              Motor Vehicles
  SEE  =       58.78 RSQ  = 0.8624 RHO =   0.63 Obser  =  105 from 1975.100
  SEE+1 =      46.01 RBSQ = 0.8444 DW  =   0.73 DoFree =   92 to   2001.100
  MAPE =        5.24
    Variable name        Reg-Coef  Mexval  Elas   NorRes     Mean   Beta
  0 cdmvRpc            - - - - - - - - - - - - - - - -      892.40 - - -
  1 intercept            17.11983    0.0    0.02    7.11      1.00
  2 dyRpc                 0.11654    3.8    0.01    7.11    103.54  0.107
  3 dyRpc[1]              0.14360    8.7    0.02    7.10    100.99  0.135
  4 dyRpc[2]              0.14234   17.3    0.02    7.03    101.25  0.134
  5 dyRpc[3]              0.13794   18.7    0.02    6.91    100.31  0.131
  6 dyRpc[4]              0.13611   19.3    0.01    6.71     95.56  0.134
  7 dyRpc[5]              0.14039   22.0    0.01    6.39     94.02  0.139
  8 dyRpc[6]              0.14189   23.6    0.01    6.00     90.14  0.143
  9 dyRpc[7]              0.13654   21.3    0.01    5.57     87.86  0.139
 10 dyRpc[8]              0.11538   15.1    0.01    5.15     85.13  0.118
 11 dyRpc[9]              0.06964    8.3    0.01    4.65     88.50  0.073
 12 rtbXypc[1]           -0.06928    9.1   -0.02    3.09    277.13 -0.185
 13 mvWearpc              0.99058   75.9    0.87    1.00    780.03  0.647


id cdmvR = cdmvRpc*pop
```

The fit is shown below in the graph on the left. The graph on the right is to help interpret the results. It shows how expenditures would respond if, after a long period of being constant, income were to rise by $1.00 and then remain constant at that new value. During the period of constant income, expenditures on motor vehicles would have reached a constant, equilibrium level. In the first quarter of the income rise, motor vehicle expenditures would rise by $1165. In the second quarter they would be $.1436 (the coefficient on dyRpc[1], which would be 1.00 in that quarter) above the pre-rise equilibrium.
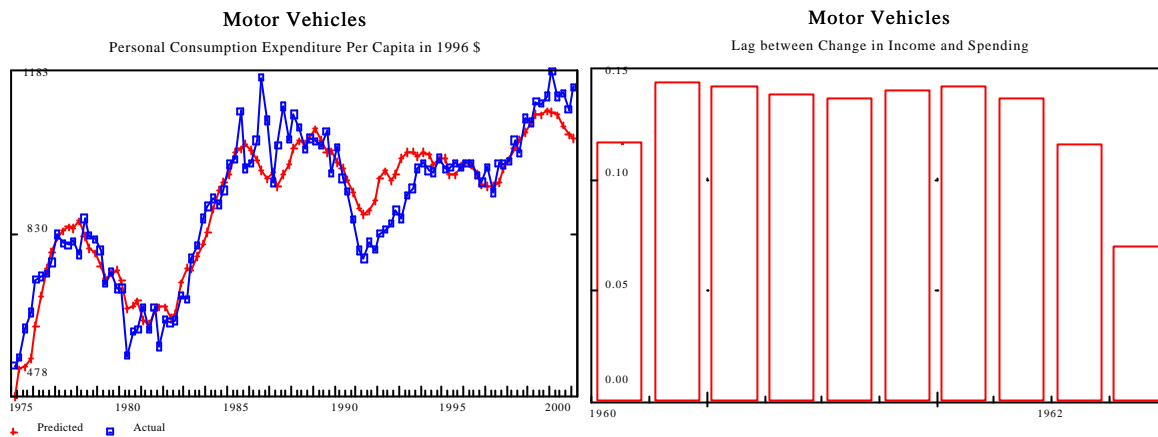
This sort of response will characterize many of our equations. We won't graph the others, but it is important for the reader to visualize these responses. This tendency of consumers to "go on a spree" of automobile buying after an increase in income is both very understandable — the increase in income allows them to borrow the money to buy the cars — and very much a generator of cycles in the economy. Actually, in this particular case, we have somewhat oversimplified the response, because, four quarters after the response of expenditures begins, the replacement response through the mvWear term begins, faintly at first, then producing a damped wave of expenditures as the initial purchases are replaced.

The fit of the automobile equation is surprisingly good, given the volatile nature of the series. Besides the strong and long transient response to increases in income and the replacement wave, the equation is noteworthy for its negative (theoretically correct) response to interest rates. Just how large a response is this? Perhaps the best answer here is given by the beta coefficient of -.185. That is to say, as the interest rate variable moves by 1.0 standard deviations, the dependent variable moves by .185 of its standard deviations. Another way to look at this question is to ask how much would a one point drop in the interest rate, say from 6 percent to 5 percent, increase expenditures on motor vehicles. At the mean value of yRpc, 18242, the answer is $12.64 per person per year. The swing from the low point of the dependent variable in 1980 to its high point in 1986, was $600, so the sensitivity to interest rates, while not negligible, is not very important.

For **Interest paid by consumers to business**, the dependent variable is expressed as a percent of disposable income. The most important explanatory variable tries to capture the interest payments on past automobile purchases. It is assumed that the loans are paid off at the rate of about 9 percent per quarter, so that about 35 percent is paid off in the first year. The outstanding amount, if all automobiles are bought with loans, is called *autfi* (automotive financing.) The interest on this amount at the Treasury bill rate (*rtb*) is called *autfir*. If the interest rate charged is *rtb+a,* then the payments should be *a*autfi + autfir.* If all automobiles and nothing else were financed, the coefficient on *autfir* should be 1.0. In the equation as estimated, both these variables are expressed as percent of disposable income, *autfin* and *autfis*, respectively. The coefficient on *autfis* comes out close to the expected 1.0, while the value of *a,* the coefficient of *autfin*, emerges as .01478, so the financing rate appears to be less about 1.5 above the Treasury bill rate, less than I

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would have expected. Notice the large values of Beta for *autfis*; the dependent variable is quite sensitive to it.

The other important variable is the exponentially-weighted average — created with the @cum function — of recent values of the savings rate. Its justification is that one way that people can save is by paying off debt on which they are paying interest. It should also be pointed out that interest payments on debt other than automotive, in so far as they are a constant fraction of disposable income, are absorbed into the intercept of the equation.   The last variable, the rate of change of the money supply, was intended to indicate the ease of getting loans. It did not prove particularly successful.

piipcb - Interest Paid by Consumers to Business
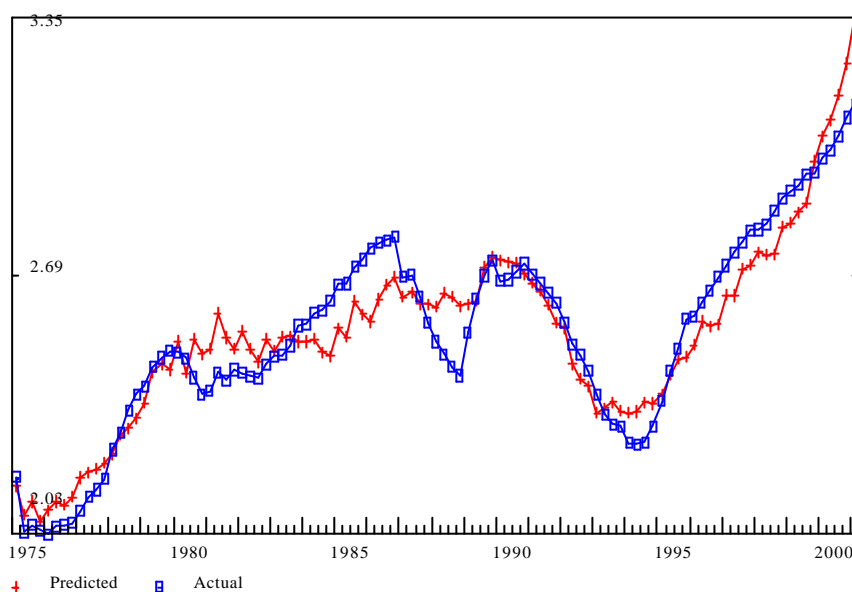

```
title piipcb - Interest Paid by Consumers to Business

#  shipcb is share of interest in disposable income less savings and transfers
fex shipcb =  100.*piipcb/pidisa

# autfi is a consumption of motor vehicles bucket with a spill of 0.09
f autfi  =  @cum(autfi ,.25*cdmv,.09)
f autfin = 100.*autfi/pidisa

f autfir = @cum(autfir,.0025*rtb*cdmv,.09)
f autfis = 100.*autfir/pidisa

#f odurfir = @cum(odurfir,.0025*rtb*(cd -cdmv),.09)
#f odurfis = 100.*odurfir/pidisa


#  savrat is the savings rate
f savrat = 100.*(pisav/pidisa)
#  b1sr is a savings rate bucket with a spill rate of 0.12
```

```
f b1sr   = @cum(b1sr,savrat,.12)
f dm1    = (m1 - m1[1])/m1[1]

:              piipcb - Interest Paid by Consumers to Business
  SEE   =      0.08 RSQ  = 0.8958 RHO =   0.87 Obser  =  105 from 1975.100
  SEE+1 =      0.04 RBSQ = 0.8916 DW  =   0.25 DoFree =  100 to    2001.100
  MAPE  =      2.66
    Variable name          Reg-Coef Mexval  Elas   NorRes     Mean    Beta
  0 shipcb               - - - - - - - - - - - - - -    2.54 - - -
  1 intercept             2.81008   130.5   1.11    9.59     1.00
  2 autfin                0.01478     0.9   0.07    9.43    11.61   0.043
  3 autfis                1.02139   114.3   0.32    9.43     0.79   0.832
  4 b1sr                 -0.01912   196.8  -0.51    1.16    67.33  -1.332
  5 dm1                   2.82571     7.5   0.02    1.00     0.01   0.154

id piipcb = 0.01*shipcb*pidisa
```

At last we are ready for the equation with the largest dependent variable in the model, **Personal consumption expenditures**. It is estimated in per capita terms, and the most important explanatory variable is certainly disposable income per capita and its first differences. Notice that the signs on the first difference terms are all negative. Instead of the splurge effect which we saw in the case of automobiles, there is a very gradual increase in spending to the level justified by an increase in income.

Textbooks of macroeconomics usually make the savings rate — and, therefore, implicitly the consumption rate — depend on the interest rate. Our equation uses the Treasury bill rate less the expected rate of inflation, which I have called the perceived real interest rate. (The actual rate of inflation is not known until after the end of a quarter, so the expected rate may be more relevant for behavior.) To make the amplitude of its fluctuations grow with the growth of the dependent variable, it has been multiplied by real disposable income per capita to make the variable *rtbexXdi*. It has the expected negative sign, but not much importance — as indicated by its mexval — relative to the other variables which never seem to get mentioned in the textbooks.

Savings in the form of automobiles, *sautos*, is the excess of spending on motor vehicles over an estimate of their wearout. Theoretically, its coefficient should be 1.0. It came out at .755, a quite satisfactory value for such a theoretically constructed variable. Its large mexval indicates its considerable importance.

Interest paid by persons to business, called *piipcbRpc* after converting it to constant price, per capita terms, also came out with the expected negative sign. Its value indicates that about 40 percent of an increase in these interest payments will come out of consumption while 60 percent will be paid by reducing savings.

Inflation, as we know, influences interest rates and, therefore, interest income of persons. But a savvy investor will recognize that if he spends all his interest in times of rapid inflation, the real value of his interest-yielding assets will shrink. To keep up the value of his investment, he must save the fraction of his interest receipts due to inflation. The variable *intsavpcR* is an attempt to measure this amount in real terms per capita. Theoretically, its coefficient should be -1; it comes out at about -

.46, a satisfactory value for a variable whose relevance depends on very conscious consumers. This variable has a profound influence on the macroeconomic properties of the model. For example, if money supply is increased and interest rates lowered, investment is stimulated, unemployment is reduced, and inflation picks up. But as soon as it does, this variable causes an increase in savings and a reduction in consumer spending, which offsets the rise in investment. Thus, monetary policy in a model with this effect is apt to prove a weak instrument. Since the effect is both intuitively evident and quantitatively important, it is surprising that it seems to have gone unnoticed in macroeconomic textbooks.

Contributions for social insurance, even the employee's half of social security, is deducted before reaching Personal income in the NIPA. It would not be irrational, however, for consumers to consider that these contributions are, in fact, a form of saving which substitutes for their private saving. We have included the *consipcR* variable to allow for this possibility. It appears that consumers consider that these contributions are a good substitute for saving.

Unemployment may have an influence on consumption. The unemployed are likely to spend a very large fraction of their income, so, given income, we would expect spending to be high when unemployment is high. In the reverse direction, when unemployment is exceptionally low, people may recognize that times are exceptionally good and take the opportunity to increase their savings. This effect could be represented either by u, the unemployment rate, or by its reciprocal, $ur = 1/u$. The simple u gives the better mexval, 4.1 compared to 1.3, but the historical simulation is strikingly better with ur. Without either, unemployment goes decidely negative in the mid 1980's; with u, it still goes slightly negative; with ur it stays above 2 percent. Thus, this variable – of little importance in the fit of the equation – is essential to the performance of the overall model.

Last but certainly not least, we come to the real stock market value per capita, *sp500Rpc*. It is the Standard and Poor's index of 500 stocks, *sp500*, deflated by the GDP deflator and divided by population. The graph on the left below shows that this value for 1975.1 - 2001.1; Between 1975 and 1985, there was essentially no growth; between 1985 and 1995, it doubled, and then doubled again in the next two years. This sort of growth makes consumers with assets in the stock market feel wealthy. Do they spend accordingly? Indeed they do, as we see from the results, where this variable has a mexval of 67.7. the variable variable increased by 3000 between 1995.1 and 2000.3, thus increasing consumption per capita by \$1330 (= 3000*.44355). During the same period, real savings per capita fell by \$1416. Thus, nearly all of this much-publicized decline in saving may be explained by spending based on the rise in the stock market. Because of the lags, at the time of this writing it remains to be seen whether the recent decline in the stock market will, indeed, reduce spending. We will return later to the question of explaining the stock market variable itself.

The combination of all these variables gives a virtually perfect fit to personal consumption. Given the number of explanatory variables we have used, what is more remarkable is that there was enough variability in the data to identify reasonable effects for all the variables. When the equation was estimated over the period 1980.1 - 1994.1, however, no effect was found for the stock market variable. It becomes important only in the last four years.

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sp500Rpc -- Real Value of S&P 500 per capita
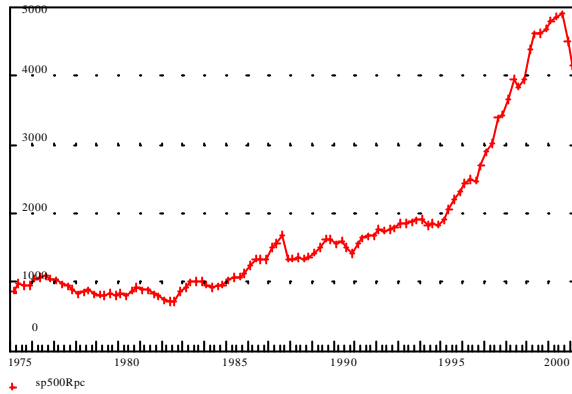
Personal Consumption per capita
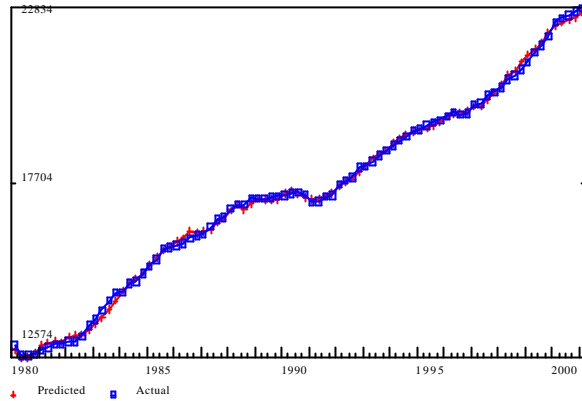
```
ti Personal Consumption per capita
fex cRpc = cR/pop

#Disposable Income per Capita
fex pidisaR = pidisa/gdpD
f yRpc = pidisaR/pop
f dyRpc = yRpc - yRpc[1]


# Interest necessary to maintain real value of assets
# First get inflex, expected inflation
fex lgdpD = 100.*@log(gdpD)
fex infl = lgdpD - lgdpD[4]
f rtbReal = rtb - infl
fex ub10 = @cum(ub10,1.0,.10)
freq ub10  4
# inflex is expected inflation
f inflex = @cum(cinfl,infl[1],.10)/ub10
f intsavRpc = (inflex/(rtb+3.))*npini/(gdpD*pop)


# Stock Market
f sp500Rpc = sp500/(gdpD*pop)
f dsp500Rpc = sp500Rpc - sp500Rpc[1]


# Perceived real interest rate
f rtbexXdi = (rtb -inflex)*yRpc
# Contributions for Social Insurance
f consiRpc = nconsi/(gdpD*pop)


# savings in autos
f sauto = cdmvRpc - mvWearpc


# Interest paid by consumers to business
f piipcbRpc = piipcb/(gdpD*pop)
f rtbXyRpc = (rtb - 5.0)*yRpc[1]


# Unemployment rate
fex u = 100.*(lfc-emp)/lfc
# Unemployment reciprocal
f ur = 1./u
sma 1000 a3 a7 1
:                   Personal Consumption per capita
  SEE   =       99.59 RSQ  = 0.9987 RHO =   0.67 Obser  =   85 from 1980.100
```

```
    SEE+1 =        74.61 RBSQ  = 0.9985 DW  =   0.65 DoFree =   69 to    2001.100
    MAPE  =         0.49
      Variable name           Reg-Coef  Mexval  Elas   NorRes     Mean  Beta
     0 cRpc                    - - - - - - - - - - - - - - -   17419.94  - - -
     1 intercept              785.42866     1.1   0.05   792.49      1.00
     2 yRpc                     0.77579    31.8   0.86     9.50  19284.28  0.711
     3 dyRpc                   -0.39068     8.5  -0.00     9.25    104.29 -0.018
     4 dyRpc[1]                -0.36579    10.9  -0.00     9.09    105.69 -0.017
     5 dyRpc[2]                -0.31072     8.3  -0.00     9.06    108.05 -0.014
     6 dyRpc[3]                -0.28662     6.3  -0.00     9.03    107.30 -0.013
     7 dyRpc[4]                -0.28233     7.0  -0.00     8.93    104.80 -0.013
     8 consiRpc                 1.04683     1.3   0.11     8.93   1831.03  0.123
     9 sp500Rpc[1]              0.45974    68.2   0.05     2.34   1933.03  0.190
    10 dsp500Rpc[1]            -0.44731     9.8  -0.00     2.23     43.17 -0.018
    11 dsp500Rpc[2]            -0.34019     4.0  -0.00     2.22     48.13 -0.013
    12 sauto                    0.72818     9.3   0.00     1.55    118.89  0.027
    13 piipcbRpc               -0.24243     0.1  -0.01     1.29    507.34 -0.008
    14 intsavRpc               -0.48752    10.1  -0.03     1.09   1151.80 -0.044
    15 rtbexXdi                -0.00161     1.5  -0.01     1.03  55178.74 -0.015
    16 ur                   -1417.29423     1.3  -0.01     1.00      0.17 -0.020
id cR = cRpc*pop
id c = cR*gdpD
```

In a conventional textbook on econometric methods, there is sure to be a chapter on simultaneous equation methods, methods for estimation when the dependent variable of an equation may influence one of the dependent variables. The essence of the problem is that, even if we know exactly the structure of the equations that describe the economy but they have random errors which we cannot observe, we may not get unbiased or even consistent estimates of the coefficients by applying least squares to the data. That is, even if we had an infinite number of observations, our estimates of the coefficients would not be right. The problem arises because, through another equation in the simultaneous system, an explanatory variable may be correlated with the error term in the equation in which it is an independent variable. The prime example is precisely income in the consumption equation, for if there is a large positive "disturbance" to the equation – a consumption spree – income will go up. This "backdoor" relationship between consumption and income would make the estimate of the coefficient on income tend to be too large. This problem, know as *simultaneous equation bias*, or less correctly as *least squares bias,* was a major theoretical concern in the early days of econometric theory, and various ways were devised to avoid it. Some of these were known as *full-information maximum likelihood, limited-information maximum likelihood, two-stage least squares,* and *instrumental variables.*

How important is this effect likely to be? One way to answer -- the instrumental variable approach -- is to regress disposable income, *pidisa* in our case, on other variables not dependent on *c* in the same period, and then to use the predicted value from this regression instead of the actual *pidisa* in estimating the equation. The coefficient should be a lower bound of the true value. I regressed *pidisa* on itself lagged once and on current values of *v, g, fe,* and *fi.* When the predicted values of this equation were used in place of *pidisa* in the consumption equation, the coefficient on *yRpc* dropped from .77 to .65 and the intercept rose to offset this drop. Other coefficients were little affected. Thus, it does not appear that the .75 is a serious overestimate. The .65, however, should

not in my view be used in place of the .75 for it suffers from its own biases arising from the fact that the *pidisa* on which it is based does not include the effects on income of numerous other factors such as tax changes and productivity fluctuations. This finding of rather minimal importance for the simultaneous equation problem is in line with the general experience of practical model builders working with quarterly data. In all other equations, we will use least squares with little concern about the problem.

(I also did a calculation I have not seen advocated in a textbook. I regressed over the period 1975.1 - 2001.1 *pidisa* on the variables mentioned above plus *c*, and then used the residuals of this equation, which are correlated with *pidisa* but not with *c*, along with the previously mentioned instrumental variables in the regression for *cRpc*. The coefficient on *yRpc* came out at .71, midway between the .77 ordinary-least-squares estimate and the .65 of the first instrumental variables estimate. This estimate may be the best available to us, but it is so little different from the ordinary-least-squares estimate that it hardly seems worth the trouble to use it. We shall return to the value of this coefficient in the following chapter on optimization.)
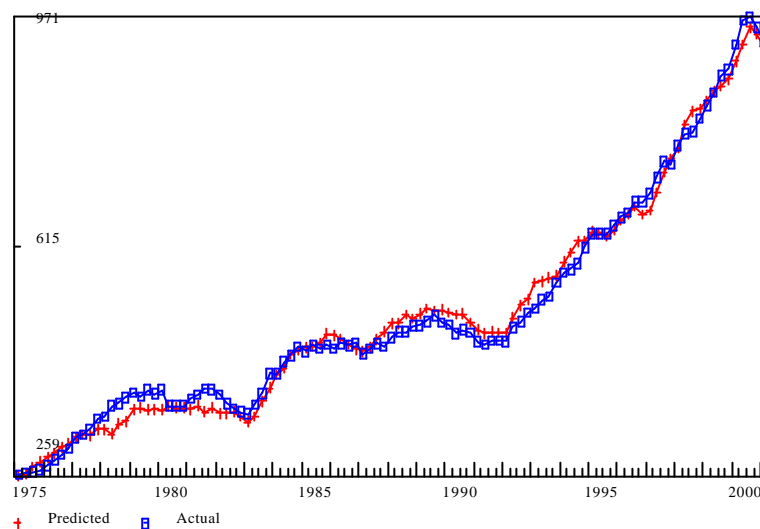
Investment

Gross private domestic investment in Quest is treated in the four major parts available in even the aggregated version of the NIPA: Producers' durable equipment, Non-residential construction, Residential construction, and Change in business inventories.

The first and largest is investment in **Producers' durable equipment**. The term for replacement is familiar from the equation for investment in AMI. Two small changes have been made in the variable whose first differences are used to indicate the need for expansion investment: (1) it is gross *private* product, since it is being used to explain *private* investment, and (2) it is the @peak function of this variable. The @peak function is the highest value which the variable has ever had up to and including the present. The use of the @peak function makes little difference in estimating the equation, but it makes the model more stable, since the first difference terms cannot go negative in a downturn. Notice the strong positive transient or "splurge" effect of an increase in output. This behavior makes equipment investment one of the primary generators of cycles in the economy.

The real interest rate used is the difference between the Treasury bill rate and the rate of inflation in the GDP deflator. Its mean value is about 2.0, and this mean has been subtracted so that the variable just shows the fluctuations about the mean. This variable is then multiplied by the replacement term divided by its mean, so the amplitude of the fluctuations in the variable will grow more or less in line with the growth of the dependent variable. A change of one percentage point will, when replacement is at its mean, change this variable by one unit. Thus, a reduction of the real interest rate by one percentage point, say from 3 to 2 — a big change -- will increase investment by about $8 billion (2.00+2.94+2.62 = 7.56), or about 1.5 percent of its mean value over this period. For an effect that dominates macroeconomics books (via the IS curve), its quantitative importance is embarrassingly small.

The stock market variable is relevant to this equation because it affects the perceived cost of funds to firms. Firms can raise funds for capital investment by selling additional shares, but the profits must then be spread over a larger number of shares and, if a particular individual or group exercises control over the company through the number of shares it holds, it may well be reluctant to see that control weakened by issuing new shares to outsiders. These objections, however, may be overcome if the stock price is high so that a lot of capital is raised with little dilution of ownership. While this effect has long been recognized as possible, it has become practically important only since 1995. Our variable, sp500R, rose by 700 between 1996.1 and its peak in 2000.3. According to our equation, this rise adds $78 billion (= .1114*700) to annual investment, and the transient effect may be even larger. Without the use of this variable, the equation fits fine up through 1994, but then falls substantially short.

Equipment Investment

Predicted   Actual

```
ti Equipment Investment
f gppR = gdpR - gdpg/gdpD
f pgppR = @peak(pgppR, gppR, .00)
f d = pgppR - pgppR[1]
f ub05 = @cum(ub05, 1.0, .05)
f repEq = @cum(stockEq, vfnreR[4], .05)/ub05
# Compute real interest rate
fex lgdpD = 100.*@log(gdpD)
fex infl = lgdpD - lgdpD[4]
fex ub10 = @cum(ub10, 1., .10)
# inflex is expected inflation
fex inflex = @cum(cinfl, infl[1], .10)/ub10
f rtbReal = rtb - infl
f rrXrepe = (rtbReal-2.)*(repEq/400.)

f sp500R = sp500/gdpD
f dsp500R = sp500R - sp500R[1]

con 10000 1 = a2
sma 1000 a3 a13 1
sma 1 a14 a16 1
sma 100 a18 a24 2
:                          Equipment Investment
```

```
SEE    =       18.34 RSQ  = 0.9891 RHO =   0.84 Obser  =  105 from 1975.100
SEE+1  =       10.10 RBSQ = 0.9860 DW  =   0.33 DoFree =   81 to   2001.100
MAPE   =        3.22
     Variable name           Reg-Coef  Mexval   Elas    NorRes     Mean   Beta
  0 vfnreR                   - - - - - - - - - - - - - - - -      510.68 - - -
  1 intercept                -4.27195     0.2  -0.01   115.16       1.00
  2 repEq                     0.96701   453.7   0.77     9.52     404.92  0.626
  3 d[1]                      0.25219    11.8   0.02     8.66      44.46  0.056
  4 d[2]                      0.23357    21.6   0.02     7.45      44.18  0.052
  5 d[3]                      0.21093    17.4   0.02     6.63      44.16  0.047
  6 d[4]                      0.19452    16.0   0.02     5.95      43.03  0.043
  7 d[5]                      0.17904    15.0   0.01     5.44      41.96  0.039
  8 d[6]                      0.15704    11.6   0.01     5.11      40.53  0.033
  9 d[7]                      0.14160     9.0   0.01     4.87      39.67  0.030
 10 d[8]                      0.12513     7.1   0.01     4.69      39.57  0.026
 11 d[9]                      0.10353     5.1   0.01     4.54      39.52  0.022
 12 d[10]                     0.07815     2.8   0.01     4.43      38.95  0.016
 13 d[11]                     0.04705     1.2   0.00     4.32      39.32  0.010
 14 rrXrepe[7]               -1.99817     1.4  -0.00     3.50       0.57 -0.025
 15 rrXrepe[8]               -2.93373     8.3  -0.00     3.17       0.56 -0.037
 16 rrXrepe[9]               -2.62119     4.5  -0.00     2.91       0.54 -0.033
 17 sp500R[1]                 0.11140    22.9   0.10     1.06     443.23  0.203
 18 dsp500R[1]                0.01580     0.0   0.00     1.05      10.04  0.003
 19 dsp500R[2]                0.06725     0.5   0.00     1.05      10.64  0.010
 20 dsp500R[3]                0.09055     0.9   0.00     1.05      10.33  0.014
 21 dsp500R[4]                0.11208     1.4   0.00     1.04       9.88  0.017
 22 dsp500R[5]                0.13665     2.0   0.00     1.03       9.38  0.021
 23 dsp500R[6]                0.10622     1.2   0.00     1.00       8.99  0.016
 24 dsp500R[7]                0.04871     0.2   0.00     1.00       8.60  0.008


save off
gname vfnreR
gr *
ti Gross Equipment Investment and Replacement
gname gir
gr repEq vfnreR
catch off
```

Investment in **Non-residential construction** — stores, office buildings, industrial plants, pipelines, churches, hospitals, airports, parking lots, and so on — is one of the hardest series to explain. Even the booming economy of the late 1990's did not bring it back to the levels it reached in the recession years of the early 1980's. Our equation is motivated by the idea that investment is proportional to the difference between the desired stock and the actual stock of structures, and that the desired stock is a linear function of the real Gross private product, gppR. Thus, the basic idea is that

$$vfnrsR = ?(a + b *gppR - StockSt)$$

where *vfnrsR* is real investment in non-residential construction, and *StockSt* is the stock of those structures. Several depreciation rates have been tried for calculating the stock of structures without much effect on the fit of the equation. One percent per quarter was chosen. By introducing lagged values of the first difference of *gppR*, the desired level of the stock is allowed to rise gradually following an increase in *gppR*.

The natural variable to add next is some sort of interest rate. These all had positive — wrong — signs with lags of three years or less. The real rate with a lag of 16 quarters has been left more or

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less as a reminder of the perverse results with shorter lags. This strong positive relation with interest rates suggested using interest income, which, indeed proved somewhat helpful. The reasoning is that persons with significant amounts of interest income might be likely to investment in real estates.

The rates of change of the stock market value variable — but not its level — also proved helpful. This variable may be measuring optimism about the future of the economy.

Finally, a special dummy variable was introduced for the period between the 1981 and the 1986 tax acts. The 1981 act allowed passive partners in real estate development (as well as active partners) to count paper depreciation at double declining balance rates against their ordinary income. Investors looking for tax shelters poured billions of dollars into non-residential construction. The 1986 act repealed this provision for non-residential construction. It did not even "grandfather" in the buildings that had been built while the 1981 act was in force. Thus, many investors who had bought tax shelters found themselves with more or less worthless holdings. Though the 1986 act was not passed until the middle of the year, its passage was anticipated, and investment was cut back for the beginning of the year.

vfnrsR - Non-residential Structures
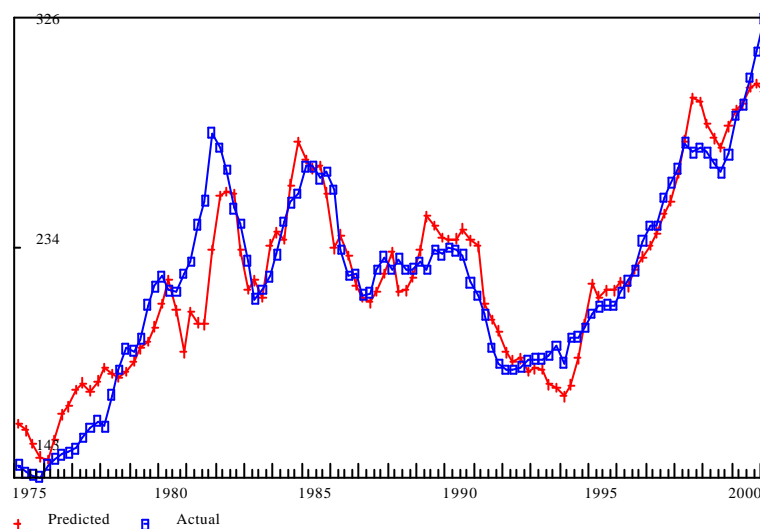
```
ti vfnrsR - Non-residential Structures
fex gdpgR = gdpg/gdpD
f gppR = gdpR - gdpgR
f pgppR = @peak(pgppR,gppR,.00)
f d = pgppR - pgppR[1]
f ub01 = @cum(ub01,1.,.01)
f StockSt = 100.* @cum(cumSt,0.25*vfnrsR[4],.01)/ub01
# Compute real interest rate
fex lgdpD = 100.*@log(gdpD)
fex infl = lgdpD - lgdpD[4]
fex ub10 = @cum(ub10,1.,.10)
# inflex is expected inflation
fex inflex = @cum(cinfl,infl[1],.10)/ub10
```

```
fex rtbReal = rtb - infl
f npiniR= npini/gdpD
# 1987 Tax Act
# The stimulus of the 1981 tax act is here shown as beginning in 1982.
# The 1986 repeal of the tax shelters created by the 1981 act was retro-
# active to the beginning of 1986, and this fact was apparently anticipated.
fex taxacts = 0
update taxacts
1982.1  1 1 1 1  1 1 1 1  1 1 1 1  1 1 1 1;
fdup sp500R = sp500/gdpD
fdup dsp500R = sp500R - sp500R[1]
```

```
:                       vfnrsR - Non-residential Structures
  SEE   =        13.93 RSQ  = 0.8759 RHO =   0.75 Obser  =  105 from 1975.100
  SEE+1 =         9.44 RBSQ = 0.8582 DW  =   0.51 DoFree =   91 to   2001.100
  MAPE  =         5.00
```

	Variable name	Reg-Coef	Mexval	Elas	NorRes	Mean	Beta
0	vfnrsR	- - - - - - - - - - - - - - - - -				221.98	- - -
1	intercept	460.45995	83.3	2.07	8.06	1.00	
2	gppR[4]	0.04231	25.4	1.01	5.28	5291.40	1.423
3	d[4]	-0.01792	0.1	-0.00	4.88	43.03	-0.018
4	StockSt[1]	-0.18779	48.3	-3.50	4.87	4133.58	-3.071
5	taxacts	33.07219	21.2	0.02	3.23	0.15	0.301
6	npiniR[1]	0.78113	18.8	2.42	1.17	689.10	4.009
7	npiniR[2]	-0.34206	3.5	-1.05	1.09	682.81	-1.771
8	rtbReal[16]	0.54464	0.3	0.01	1.09	2.09	0.043
9	dsp500R[3]	0.01933	0.0	0.00	1.08	10.33	0.013
10	dsp500R[4]	0.06997	0.6	0.00	1.07	9.88	0.047
11	dsp500R[5]	0.07541	0.7	0.00	1.06	9.38	0.051
12	dsp500R[6]	0.10387	1.2	0.00	1.03	8.99	0.071
13	dsp500R[7]	0.10612	1.2	0.00	1.00	8.60	0.073
14	dsp500R[8]	0.03896	0.2	0.00	1.00	7.93	0.026

Investment in **Residential constuction**, quite in contrast to non-residential construction, proves to be quite sensitive in the proper, negative direction to interest rates. Otherwise, the approach to the equation is similar except that a combination of disposable income and the stock market value is presumed to determine the desired stock.

Residential Construction
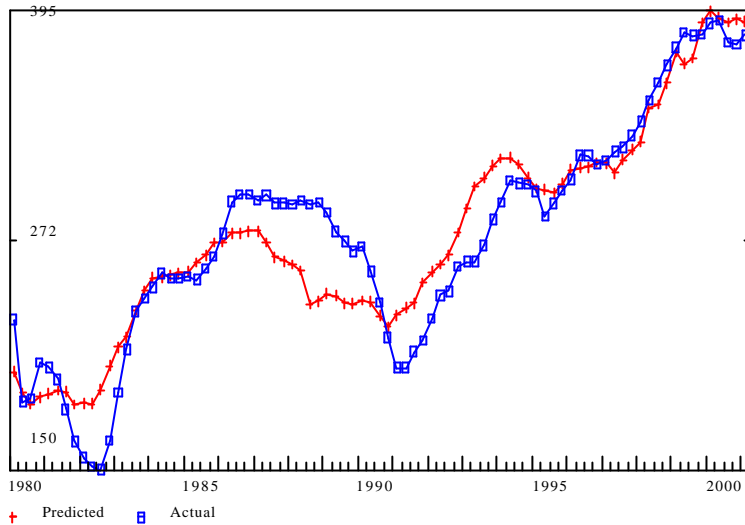

```
ti Residential Construction
fex lgdpD = 100.*@log(gdpD)
fex infl = lgdpD - lgdpD[4]
fex ub10 = @cum(ub10,1.0,.10)
freq ub10  4
# inflex is expected inflation
fex inflex = @cum(cinfl,infl[1],.10)/ub10
fex rtbex = rtb - inflex

f ub01 = @cum(ub01,1.,.01)
f StockHouse = 100.*@cum(cvfrR,0.25*vfrR[2],.01)/ub01

f pidisaR = pidisa/gdpD
f dpidisaR = pidisaR - pidisaR[1]
fdup sp500R = sp500/gdpD
sma 100 a7 a11 1
:                    Residential Construction
  SEE   =      21.19 RSQ  = 0.8739 RHO =   0.91 Obser  =   85 from 1980.100
  SEE+1 =       9.18 RBSQ = 0.8569 DW  =   0.18 DoFree =   74 to   2001.100
  MAPE  =       6.74
```

	Variable name	Reg-Coef	Mexval	Elas	NorRes	Mean	Beta
0	vfrR	- - - - - - - - - - - - - - - - -				274.55	- - -
1	intercept	996.38316	36.7	3.63	7.92	1.00	
2	pidisaR[4]	0.18933	36.6	3.26	2.23	4728.85	2.919
3	dpidisaR[4]	-0.09545	0.9	-0.01	2.18	37.36	-0.051
4	dpidisaR[5]	-0.03008	0.1	-0.00	2.11	37.26	-0.016
5	sp500R[3]	0.19678	42.3	0.34	1.78	473.54	1.022
6	StockHouse	-0.32797	30.5	-6.16	1.09	5155.30	-3.036
7	rtbex[4]	-1.63794	3.0	-0.02	1.09	2.88	-0.042
8	rtbex[5]	-1.36392	4.0	-0.01	1.08	2.86	-0.035
9	rtbex[6]	-1.07337	4.0	-0.01	1.06	2.83	-0.028
10	rtbex[7]	-0.74906	2.9	-0.01	1.04	2.80	-0.020
11	rtbex[8]	-0.38668	1.8	-0.00	1.00	2.77	-0.010

Finally, investment in **Change in business inventories** is unchanged from the AMI model but is repeated here for completeness.

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viR Change in Inventory
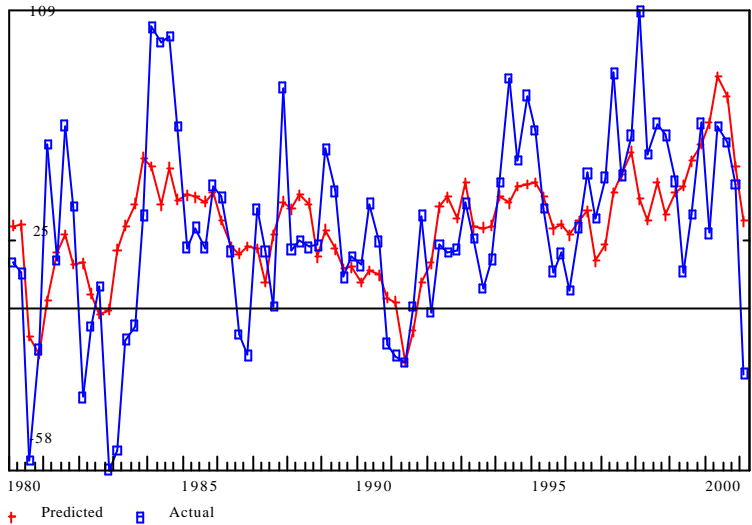
```
title viR Change in Inventory
# fs stands for "final sales"
f fsR = cR + vfR + feR + gR
f dfsR = fsR - fsR[1]
sma 1000 a1 a4 1
:                        viR Change in Inventory
  SEE   =       27.69 RSQ   = 0.3511 RHO =    0.43 Obser  =   85 from 1980.100
  SEE+1 =       25.20 RBSQ  = 0.3271 DW  =    1.14 DoFree  =   81 to   2001.100
  MAPE  =      131.86
    Variable name          Reg-Coef  Mexval  Elas   NorRes     Mean   Beta
  0 viR                   - - - - - - - - - - - - - - - - -   29.74 - - -
  1 dfsR[1]                 0.22792    11.3   0.50     1.38    64.84
  2 dfsR[2]                 0.15384    12.7   0.34     1.04    65.73  0.234
  3 dfsR[3]                 0.05849     1.7   0.13     1.01    65.14  0.090
  4 dfsR[4]                 0.02831     0.4   0.06     1.00    63.05  0.044
```

Exports, Imports, and the Terms of Trade

The natural economic variable to use in explaining imports or exports is the domestic price over the
foreign price for similar goods, the *terms of trade* for that product. Earlier versions of Quest used
a terms of trade variable computed from the overall import deflator relative to the domestic prices of
tradable final demand goods. It never worked very well and was hard to model. In this revision, I
looked at the import deflator relative to the export deflator for all the major categories of traded
goods. The graph below shows the results for three typical product groups. Clearly, there is little
or no similarity among them. There is no possibility of finding a single index to represent then all and
equally little possibility to explain such different series with similar equations using the same
macroeconomic explanatory variables. I have therefore given up on explaining and using a terms of
trade variable. Instead, we will use directly in the export and import equations the variables that
might have been used to explain terms of trade.

Alternative Terms of Trade
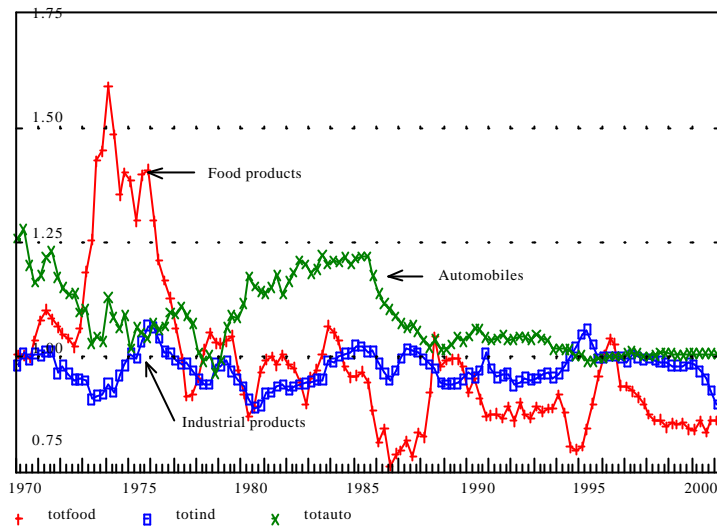
The primary variable in the explanation of **exports** is foreign demand, *fgndem*. This variable, a by-product of the Inforum International System of multisectoral models, is a combination of the real imports of the major trading partners of the United States, weighted together with their shares in U.S. exports in 1992. The dependent variable of our equation, *xRat,* is the logarithm of the ratio of our real exports to this variable. The unemployment rate enters the explanation because at times of low unemployment U.S. firms may not be able give good prices or delivery times to foreign customers, who then turn elsewhere for suppliers. Consequently, a high unemployment rate should make for a high *xRat.* Our result shows that a one percentage point increase in our unemployment rate increases our exports by over 5.0 percent. The real interest rate can be important, because at times of high interest rates, foreigners buy dollars to get the high interest, thus running up the value of the dollar and limiting U.S. exports. According to our equation, a one point increase in the real interest rate can decrease exports by 3.7 percent. A similar argument applied to the stock market. A strong market attracts foreign investors, who buy dollars to buy American stocks, thereby pushing up the dollar and making it difficult for U.S. manufacturers to compete abroad. The variable used for the stock market is the S&P 500 index relative to nominal GDP; this variable has roughly the same value today as it did forty years ago, and thus appears to be stationary. The final variable used, *d80*, is a dummy which assumes positive values only in the period from 1979 to 1982. During this period, the fit of the equation without *d80* had large positive errors in that period. I was unable to find a variable to eliminate these errors but added *d80* so that simulations of the model beginning with 1980 would not start off with large errors in exports. This equation was fit over a rather long period because fitting from 1980 forward gave a wrong sign on real interest variable; the experience of the 1970's is necessary for the program to be able to find the logically correct relationship.

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Exports Relative to Foreign Demand
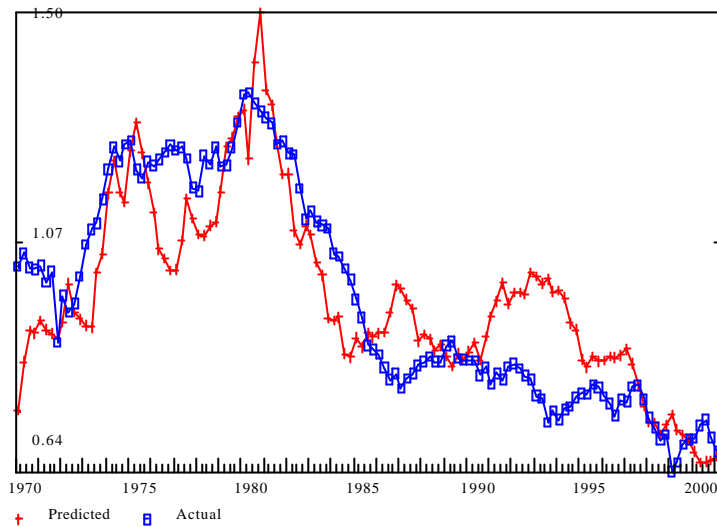

```
ti Exports Relative to Foreign Demand
fex xRat = @log(feR/fgndem)

f pgdp = fe+vf+c
f sprat = sp500/pgdp
fex d80 = 0
update d80
1979.1 .2 .4 .6  .8
1980.1  1. 1. 1. 1.  1. 1. 1. 1.  .8 .6 .4 .2
:                  Exports Relative to Foreign Demand
  SEE   =       0.11 RSQ   = 0.6982 RHO =   0.89 Obser  =  125 from 1970.100
  SEE+1 =       0.06 RBSQ  = 0.6882 DW  =   0.23 DoFree =  120 to   2001.100
  MAPE  =       9.57
    Variable name           Reg-Coef  Mexval  Elas   NorRes    Mean   Beta
  0 xRat                 - - - - - - - - - - - - - - - - -     0.97 - - -
  1 intercept               0.80538    25.2   0.83    3.31     1.00
  2 unemp[1]                0.05037     9.1   0.33    2.65     6.32  0.356
  3 rtbReal[1]             -0.03674    33.0  -0.08    2.07     2.15 -0.528
  4 sprat[1]               -1.44963     3.2  -0.11    1.73     0.08 -0.210
  5 d80                     0.39038    31.6   0.04    1.00     0.10  0.521

id feR = @exp(xRat)*fgndem
```

The equation for **imports** is similar but uses components of aggregate demand, consumption, investment, and exports in place of the foreign demand variable. Because these different demand components may have different import content, the shares of two of them, exports and investment, in the total are used as explanatory variables and prove to have positive effects, that is, they are more import-intensive than is the third component, consumption. The the stock market index is included here for the same reason as it was included in the export equation, though with the opposite expected sign.

Import Ratio to Private Demand
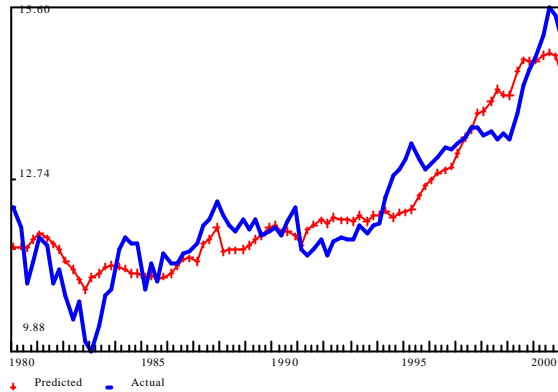
```
ti Import Ratio to Private Demand
f pgdp = fe+vf+c
f imprat = 100.*fi/pgdp
f ferat = fe/pgdp
f vfrat = vf/pgdp
f sprat = sp500/pgdp
:                      Import Ratio to Private Demand
  SEE   =       0.47 RSQ  = 0.8486 RHO =   0.82 Obser  =   85 from 1980.100
  SEE+1 =       0.27 RBSQ = 0.8430 DW  =   0.35 DoFree =   81 to   2001.100
  MAPE  =       3.21
    Variable name          Reg-Coef  Mexval  Elas   NorRes      Mean   Beta
  0 imprat              - - - - - - - - - - - - - - - - -      12.24  - - -
  1 intercept               5.98204    16.3   0.49     6.60      1.00
  2 ferat                  21.51918     8.5   0.18     4.19      0.10  0.230
  3 vfrat                  10.07827     3.3   0.14     3.75      0.17  0.119
  4 sprat[1]               29.74946    93.7   0.18     1.00      0.08  0.793


id fi = imprat*pgdp/100.
id fiR = fi/gdpD
```

Productivity, Employment, and Unemployment

As an exercise in Chapter 3, we added to the original AMI model and equation for employment
which simply regressed employment on real Gross domestic product. Implicitly, this made all the
growth in productivity depend on the growth in real GDP. Here we need to examine that growth
more closely.

First of all, we need to note that our employment variable, *emp*, is civilian employment and does not
count members of the military. As far as I can see, people in the military do not exist for the Bureau
of Labor Statistics (BLS). All of the familiar data on labor force, employment, and unemployment
statistics are for civilians only. I have been unable to find a BLS series on military employment. The
right way to handle this problem would be to construct a quarterly series on military employment
and use it to convert all of the BLS series to a total labor force basis. The difficulty of maintaining
this series, however, and the loss of comparability with familiar BLS statistics has led me to go into
the other direction, namely, to deduct real compensation of the military – which is readily available

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in the NIPA – from gdpR to get gdpcR, real civilian GDP and to use it to explain civilian employment.

Our dependent variable will therefore be the logarithm of gross civilian labor productivity, real civilian GDP divided by civilian employment. Regressed simply on *time*, over the period 1980.1 - 2001.1, the coefficient on *time* is .01716, that is, 1.7 percent per year. Besides time, however, there are at least two other factors readily available which should be tried. From the investment equation, we have available the stock of equipment from which we can make up a capital-output ratio. This ratio was more volatile than the dependent variable, so it was smoothed. To avoid spurious correlation from having real GDP in the denominator of both variables, we have used only lagged values in this variable, *capouts*.

Another factor is real GDP itself. It could influence productivity by economies of scale and by the opportunities which growth gives to eliminate inefficiencies without the painful process of laying off workers. When it was introduced into the equation, it was very successful; and the coefficient on *time* fell to only .00473. There is, however, a problem with this variable, for it occurs in the numerator of the dependent variable. Thus, any random fluctuation in it will show up automatically as a similar fluctuation in productivity. Thus, if we are really looking for long-term relations, the *gdpR* variable may get too high a coefficient relative to the *time* variable. To control for this situation, the equation was run with gdpR[1] as the most recent value of this variable. The coefficient on *time* rose to .00687. We then constrained the coefficient at that value, restored the use of the current value of *gdpR*, and re-estimated the equation.

Fluctuations in productivity are explained largely by the lagged values of the percentage change in real GDP, here calculated as the first difference of the logarithm. Notice the big surge in productivity which follows an increase in real GDP. It is initally produced by existing employees simply working harder and longer and perhaps by some postponable work simply being postponed. Gradually, however, employment is brought up to the levels appropriate for the level of output. For every 1 percent increase in real GDP, we find an increase of 0.32 percent in productivity.

Labor Productivity
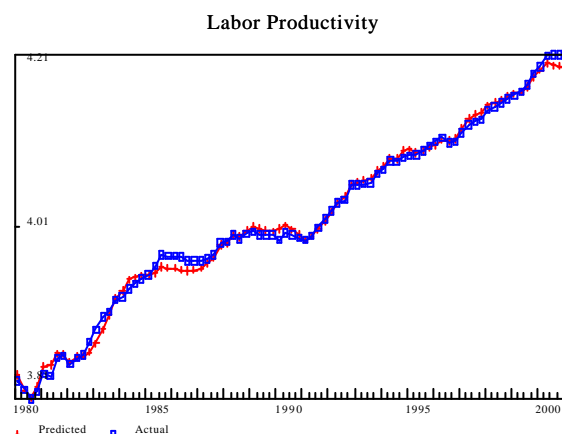
ti Labor Productivity
Military compensation in real terms

22

```
fex gfdcemR = gfdcem/gdpD
# Create Civilian GDP
f gdpcR = gdpR - gfdcemR
fex lLabProd = @log(gdpcR/emp)
f lgdpcR = @log(gdpcR)
f pcGdpcR = lgdpcR - lgdpcR[1]
fdup repEq = @cum(stockEq,vfnreR[4],.05)/ub05
f pgdpcR=@peak(pgdpcR,gdpcR,.0)
f capout = repEq/pgdpcR
f lcapouts = @log(.5*capout[1]+.3*capout[2]+.2*capout[3])
sma .001 a4 a11 1
con 100 .00623 = a2
:                         Labor Productivity
  SEE   =        0.01 RSQ  = 0.9962 RHO =   0.86 Obser =   85 from 1980.100
  SEE+1 =        0.00 RBSQ = 0.9956 DW  =   0.28 DoFree =  73 to   2001.100
  MAPE  =        0.13
    Variable name        Reg-Coef  Mexval  Elas   NorRes    Mean   Beta
  0 lLabProd           - - - - - - - - - - - - - - - - -    4.02  - - -
  1 intercept            1.18387    70.6   0.29   339.50    1.00
  2 time                 0.00622   833.8   0.05    79.70   30.75   0.357
  3 lgdpcR               0.32108   674.4   0.70     3.42    8.77   0.614
  4 pcGdpcR              0.46550    34.4   0.00     3.38    0.01   0.035
  5 pcGdpcR[1]           0.44218    58.9   0.00     3.11    0.01   0.033
  6 pcGdpcR[2]           0.41816    68.9   0.00     2.64    0.01   0.031
  7 pcGdpcR[3]           0.38969    60.3   0.00     2.21    0.01   0.029
  8 pcGdpcR[4]           0.35073    47.7   0.00     1.86    0.01   0.027
  9 pcGdpcR[5]           0.29598    36.0   0.00     1.60    0.01   0.023
 10 pcGdpcR[6]           0.22099    25.9   0.00     1.39    0.01   0.017
 11 pcGdpcR[7]           0.12084    17.1   0.00     1.09    0.01   0.009
 12 lcapouts             0.07172     4.2  -0.05     1.00   -2.72   0.021

f LabProd = @exp(lLabProd)
id emp = gdpcR/LabProd
```

With labor productivity known, **employment** is just computed by dividing real GDP by it; **unemployment** is computed by subtracting employment from the labor force.

Interest rates

The key to obtaining a somewhat satisfactory explanation of the interest rate was to use as the dependent variable the "expected" or "perceived" real interest rate — the nominal rate on 90-day Treasury bills minus the expected rate of inflation. The sole explanatory variable is the velocity of M1 together with lagged values of its first difference, and it product with time. The negative coefficient on the product of velocity and time indicates a gradual reduction in the requirements for M1. The positive signs on the first differences indicate that the immediate impact on interest rates of a change in money supply relative to GDP is substantially greater than the long-term impact. Seemingly, the financial institutions adjust to the available money supply. During an earlier period, M2 would have been the appropriate measure of money; but during the period studied here, it has little value in explaining interest rates.

Treasury Bill Rate
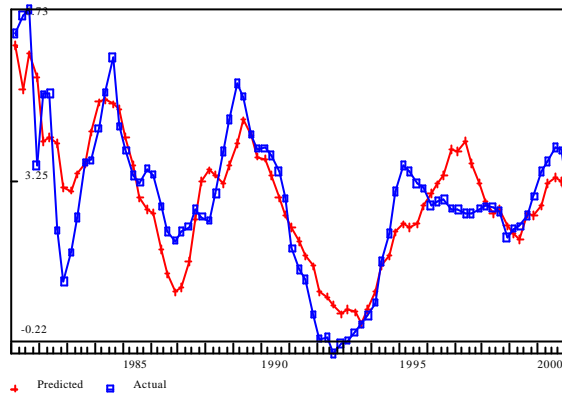
```
ti Treasury Bill Rate
f lgdpD = 100.*@log(gdpD)
f infl = lgdpD - lgdpD[4]
fex ub10 = @cum(ub10,1.0,.10)
freq ub10  4
# inflex is expected inflation
f inflex = @cum(cinfl,infl[1],.10)/ub10
fex rtbex = rtb - inflex
f v1 = gdp/m1
f dv1 = v1 - v1[1]
sma .1 a3 a7 1
:                       Treasury Bill Rate
  SEE   =       0.77 RSQ   = 0.7325 RHO =   0.61 Obser  =   81 from 1981.100
  SEE+1 =       0.61 RBSQ  = 0.7068 DW  =   0.78 DoFree =   73 to   2001.100
  MAPE  =     107.49
    Variable name         Reg-Coef  Mexval  Elas   NorRes    Mean    Beta
  0 rtbex             - - - - - - - - - - - - - - - -     2.90  - - -
  1 intercept           -1.26265     0.9   -0.44    3.74     1.00
  2 v1                   1.39110    31.1    3.43    3.39     7.16   0.720
  3 dv1                  4.00192    31.4    0.04    3.35     0.03   0.302
  4 dv1[1]               3.16590    44.4    0.03    3.16     0.03   0.240
  5 dv1[2]               2.35631    31.5    0.02    2.95     0.03   0.179
  6 dv1[3]               1.53474    14.8    0.01    2.84     0.02   0.116
  7 dv1[4]               0.74889     6.6    0.01    2.76     0.02   0.055
  8 time*v1             -0.02704    66.0   -2.10    1.00   225.70  -1.106

id rtb = rtbex + inflex
```

The Income Side of the Accounts

To understand the connections and relevance of the remaining equations, one needs to recall the basic identities of the income side of the NIPA. In the following quick review, the items for which regression equations have been developed are shown in bold. All other items are either determined either by identities or by behavioral ratios or are left exogenous.

```
#gnp — gross national product
# gnp = gdp + exports of factor income - imports of factor income
```

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```
# Net National Product
id nnp = gnp - ncca


# ninc -- National income  — from the product side
# ninc = + nnp     Net national product
#         - nibtax  Indirect business taxes
#         - nbtrp   Business transfer payments
#         - nsd     Statisticial discrepancy
#         + nsub    Subsides less surplus of gov't enterprises
#

# The alternative, income-side definition of national income.
# ninc = + niceprop Compensation of employees and Proprietor income
#         + niren    Rental income
#         + niprf    Corporate profits
#         + netint   Net interest


# pi — Personal Income
#pi   = + ninc   National income
#        - niprf  Corporate profits with IVA and CCA
#        + npdivi Personal dividend income
#        - netint Net interest
#        + npini  Personal interest income
#        - nconsi Contributions for social insurance
#        + ngtpp  Government transfer payments to persons
#        + nbtrpp Business transfer payments to persons
#        - nwald  Wage accruals less disbursements

# npini — Personal interest income
# npini = + netint   Net interest
#          + gfenip   Net interest paid by the Federal government
#          + gsenip   Net interest paid by state and local governments
#          + piipcb   Interest paid by consumers to business
```

Notice that we have two different definitions of National income, one derived from GDP and one from adding up the five types of factor income which compose it. We will compute it both ways but scale the components of the income definition to match the product definition.

In all, there are eight different items to be determined by regression: Capital consumption allowances, four components of National income, Personal dividend income, and two Net interest payments by government. One other item, Interest paid by consumers to business, has already been discussed.

Capital consumption allowances

The computation of capital consumption allowances was explained in Chapter 1. Here we are seeking just a rough approximation of this process. We divide investment into two types: equipment and structures. For each, we set up a two-bucket wear-out system. For equipment, both buckets

have a spill rate of 5 percent per quarter; for structures, both buckets have a spill rate of 1 percent per quarter.   The weights on the spill streams from the two equipment buckets are softly constrained to add to 1.0, as are the weigts on the spill streams from the two structures buckets. Finally, a variable called *disaster* allows for the exceptional capital consumption by hurricane Andrew and by the Los Angeles earthquake of 1994. The fit was extremely close.

ncca -- capital consumption allowance
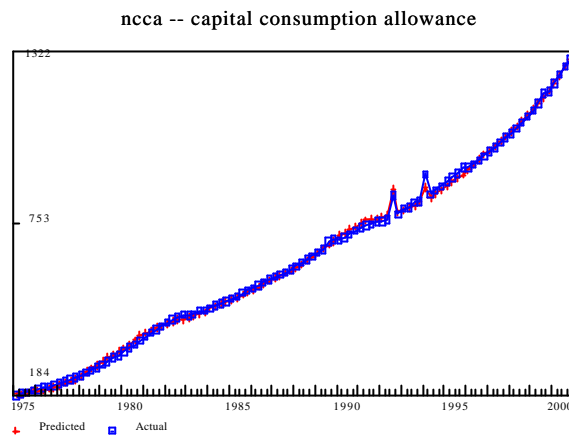
```
ti ncca -- capital consumption allowance
# Wearout of Equipment
f ub05 = @cum(ub05,1.,.05)
f repEq1R = @cum(c1vfnreR,vfnreR,.05)/ub05
f repEq2R = @cum(c2vfnreR,repEq1R,.05)/ub05

# Equipment wearout in current prices
f repEq2 = repEq2R*gdpD
f repEq1 = repEq1R*gdpD

# Wearout of Structures
f ub01 = @cum(ub01,1.,.01)
f vfsR = vfrR + vfnrsR
f repSt1R = @cum(c1vfsR,vfsR,.01)/ub01
f repSt2R = @cum(c2vfsR,repSt1R,.01)/ub01

# Structure wearout in current prices
f repSt1 = repSt1R*gdpD
f repSt2 = repSt2R*gdpD

fex disaster = 0
# disaster 92.3 = Hurricane Andrew;  94.1 = L.A. earthquake
update disaster
1992.3 1 0 0 0 0 0 .5;
con 500 1 = a2 + a3
con 500 1 = a4 + a5
r ncca = repEq1, repEq2, repSt1, repSt2,disaster
:                 ncca -- capital consumption allowance
  SEE   =       9.50 RSQ  = 0.9990 RHO =   0.68 Obser  =  105 from 1975.100
  SEE+1 =       6.96 RBSQ = 0.9990 DW  =   0.64 DoFree =   99 to   2001.100
  MAPE  =       1.36
    Variable name          Reg-Coef  Mexval  Elas   NorRes      Mean   Beta
  0 ncca                   - - - - - - - - - - - - - - - -    644.11 - - -
  1 intercept             -47.40771    68.4  -0.07   966.97      1.00
  2 repEq1                  0.86473    65.8   0.48    16.40    357.45  0.518
```

```
3 repEq2                      0.24989    4.1   0.12   12.48    301.83  0.121
4 repSt1                      0.73893   11.4   0.35    1.80    305.05  0.269
5 repSt2                      0.32906    1.8   0.12    1.76    244.37  0.093
6 disaster                   82.59208   32.7   0.00    1.00      0.01  0.029
```

Components of national income

Compensation of employees and **Proprietor income** are modeled together since our
employment variable does not separate employees from proprietors. The ratio of the combination
to total employment gives earnings per employed person, which, when put into real terms, is
regressed on labor productivity and the unemployment rate. Since employment appears in the
denominator of both the dependent and independent variables, I checked for spurious correlation
by using only lagged values of labor productivity. The coefficient on labor productivity actually rose
slightly, so there is little reason to suspect spurious correlation. The use of the unemployment
variable in this equation is a mild infraction of the rule against using a stationary variable to explain a
trended one, but percentage-wise the growth in the dependent variable has not been great in recent
years. Both the dependent variable and labor productivity are in logarithmic terms, so the
regression coefficient is an elasticity. This elasticity turns out to be slightly less than 1.0. Note that
while the mexvals on the two lagged values of the unemployment rate are both very small, the
combined effect, as seen in the Nores column, is substantial.

Real Earnings per Employed Person

Predicted   Actual
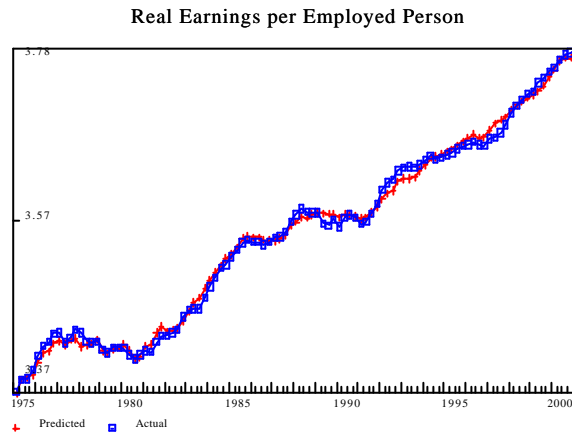
```
ti Real Earnings per Employed Person
fex lwageR = @log(((nice+niprop)/emp)/gdpD)
:                    Real Earnings per Employed Person
  SEE   =       0.01 RSQ  = 0.9954 RHO =   0.85 Obser  =  105 from 1975.100
  SEE+1 =       0.00 RBSQ = 0.9952 DW  =   0.29 DoFree =  100 to   2001.100
  MAPE  =       0.17
    Variable name          Reg-Coef  Mexval  Elas   NorRes     Mean    Beta
 0 lwageR                 - - - - - - - - - - - - - - - - -    3.56  - - -
 1 intercept               0.04575     0.8   0.01  217.20      1.00
 2 lLabProd                0.72093    18.7   0.81    1.30      3.98   0.791
 3 lLabProd[1]             0.16590     1.0   0.19    1.20      3.98   0.182
 4 u[2]                   -0.00153     0.2  -0.00    1.01      6.53  -0.020
 5 u[3]                   -0.00165     0.3  -0.00    1.00      6.55  -0.021
```

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```
f nicepro = @exp(lwageR)*emp*gdpD
save off
gname nice
gr *
catch off
```

Rental income is the smallest component of national income. It is the income of persons (not corporations) from renting out a house, a room or two in a house, or a commercial property. In particular, in includes the net rental income imputed to owner-occupants of houses, that is, the imputed space rental value less mortgage interest, taxes, and upkeep expenses. In view of this content, it is not surprising that the stock of houses should be one of the explanatory variables. It is not, however, able to explain why rental income, after decades of virtual constancy, began to rise rapidly in 1994. The only variable at our disposal to explain this takeoff is the stock market value variable. Perhaps the rise in the stock market was accompanied by a parallel rise in the value of commercial real estate, which shows up in the rental income.
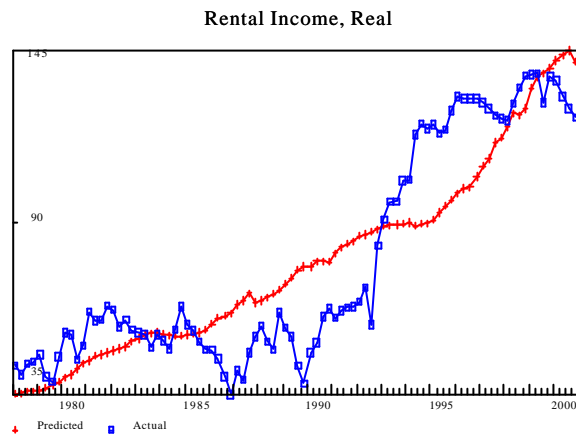
Rental Income, Real

```
ti Rental Income, Real
fdup sp500R = sp500/gdpD
f nirenR = niren/gdpD
# StockHouse defined in vfrR.reg
fex StockHouse = 100.*@cum(cvfrR,0.25*vfrR[2],.01)/ub01
r nirenR = StockHouse[8],sp500R
:                           Rental Income, Real
  SEE   =       16.51 RSQ   = 0.7747 RHO =    0.94 Obser  =   93 from 1978.100
  SEE+1 =        5.86 RBSQ  = 0.7697 DW  =    0.13 DoFree =   90 to   2001.100
  MAPE  =       20.30
    Variable name          Reg-Coef  Mexval  Elas   NorRes    Mean    Beta
  0 nirenR               - - - - - - - - - - - - - - -    78.01 - - -
  1 intercept             -82.83793     5.8  -1.06    4.44     1.00
  2 StockHouse[8]           0.02851    11.6   1.78    1.20  4879.77  0.478
  3 sp500R                  0.04495     9.6   0.28    1.00   483.20  0.432

f niren = nirenR*gdpD
```

The **Corporate profits** modeled here are the "economic" profits of the NIPA, not the "book" profits that appear in the financial reports of the corporations.   The difference lies in the two factors Inventory valuation adjustment (IVA) and Capital consumption adjustment (CCA) which eliminate

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from profits distortions caused by inflation. The equation is quite simple. It uses only real Gross private product and changes in its peak value. When real GDP rises by $1, profits rise permanently by $0.11, but in the same quarter with the rise in GDP, they go up by a stunning $0.60. Sixty percent of the increase goes into profits. Thus, profits are much more volatile than GDP. Now does this volatility amplify or dampen business cycles? Because profits are *subtracted* from GDP in the course of calculating Personal income, the volatility in profits actually makes Personal income more stable and contributes to overall economic stability.

niprfR -- Corporate Profits with IVA and CCAdj

Predicted   Actual
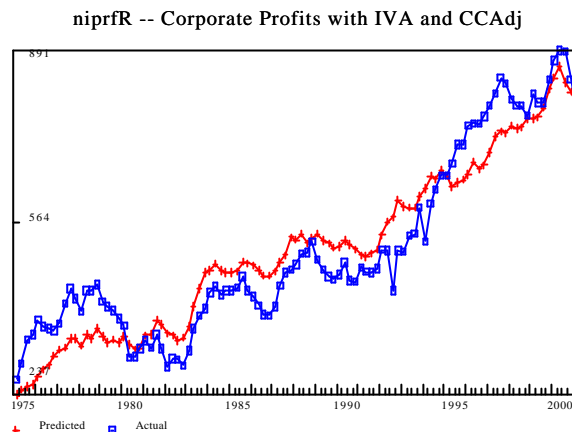
```
title niprfR -- Corporate Profits with IVA and CCAdj
fex gppR = gdpR - gdpg/gdpD
fex pgppR = @peak(pgppR,gppR,.0)
fex d = pgppR - pgppR[1]
fex niprfR = niprf/gdpD
sma 1000 a3 a6 1
:               niprfR -- Corporate Profits with IVA and CCAdj
  SEE   =      58.84 RSQ   = 0.8818 RHO =   0.93 Obser  =  105 from 1975.100
  SEE+1 =      22.49 RBSQ  = 0.8758 DW  =   0.15 DoFree =   99 to   2001.100
  MAPE  =      11.26
    Variable name          Reg-Coef  Mexval  Elas  NorRes     Mean   Beta
  0 niprfR               - - - - - - - - - - - - - - - - -   511.46 - - -
  1 intercept            -104.68758     8.6  -0.20    8.45     1.00
  2 gppR                    0.10606   126.0   1.14    1.10  5478.62  0.856
  3 d                       0.26483     1.8   0.02    1.07    44.50  0.060
  4 d[1]                    0.22196     2.8   0.02    1.04    44.46  0.050
  5 d[2]                    0.18680     1.7   0.02    1.02    44.18  0.043
  6 d[3]                    0.11682     0.8   0.01    1.00    44.16  0.027

id niprf = niprfR*gdpD
```

Net interest is all interest paid by business less interest received by business. It is modeled by estimating the debt of business and multiplying it by the interest rate. Business debt is taken to be its initial amount at the beginning of the estimation period, D_0, plus accumulated external financing since then, bdebt. This need for external financing is investment minus internal sources of funds — profits and capital consumption allowances less profits taxes and dividends paid (which are equal to dividends received plus dividends paid abroad minus dividends received from abroad). The external financing can be accomplished either by borrowing or by issuing equities. We will derive

the net interest equation as if all of the funding was by debt; we can then recognize that part of it will be financed by issuing stock. Not all debt is refinanced ever quarter, so we smooth the Treasury bill rate, producing *srtb*. Business does not necessarily pay the Treasury rate, so we add to srtb a constant, *a*, to approximate the rate it does pay. Theoretically, then, we should have

$$netint \quad = D_0*(a + srtb) + bdebt*(a+srtb).$$
$$= aD_0 + D_0*srtb + a*bdebt + bdebt*srtb$$

The fit obtained with this regression is acceptable, but the regression coefficients were not entirely consistent with expectations. The coefficient on *srtb*bdebt*, which should have been 1.0, came out when unconstrained a bit above 1.0 and was constrained down to 1.0. The coefficient on business debt, which should surely be less than .1 by the theory, came out at 0.30. But the main discrepancy is that the coefficient on *srtb*, which should be the initial debt — and therefore positive — is decidedly negative. Perhaps high interest rates induce firms to switch away from debt financing and towards equities.

netint -- Net Interest
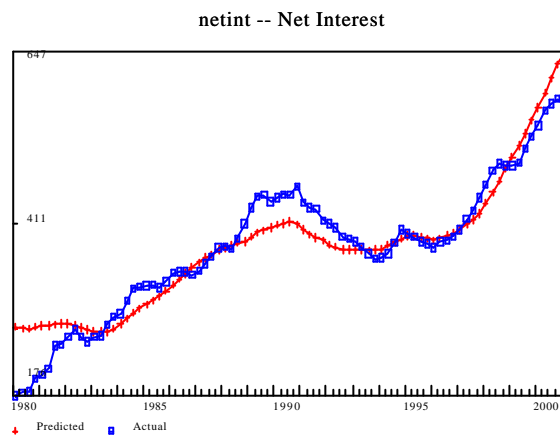
```
title netint -- Net Interest
f ub100 = @cum(ub10,1.,.1)
f srtb = 0.01*@cum(crtb,rtb[1],.1)/ub100
f bdef = v - (ncca + niprf - nictax - npdivi - gsediv + fefaci - fifaci)
# business deficit
fdates 1980.1 2005.4
f bdebt = @cum(bdebt,.25*bdef,0.0)
f rXbdebt = srtb*bdebt
# netint = bdebt(0)*(a +srtb) + bdebt*(a+srtb) ; and divide both sides by deflate
con 10000 1 = a4
```

```
:                        netint -- Net Interest
  SEE   =       31.01 RSQ   = 0.8913 RHO =    0.96 Obser  =    85 from 1980.100
  SEE+1 =       13.33 RBSQ  = 0.8873 DW  =    0.08 DoFree =    81 to    2001.100
  MAPE  =        7.33
```

Variable name	Reg-Coef	Mexval	Elas	NorRes	Mean	Beta
0 netint	- - - - - - - - - - - - - - - - -				378.63	- - -
1 intercept	333.52967	103.4	0.88	19.59	1.00	
2 srtb	-919.73313	9.9	-0.17	15.02	0.07	-0.201
3 bdebt	0.19210	75.4	0.22	11.57	434.25	0.641
4 rXbdebt	1.00750	240.1	0.07	1.00	26.33	0.160

30

```
fdates 1960.1 2005.4
```

Dividends

The most important determinant of **dividends**, not surprisingly, is profits; and most of our equation just amounts to a long distributed lag on past profits. Because appreciation of the value of stock can also substitute, in the eye of the investor, for dividends, we have also included changes in the value of the stock market, which gets the expected negative sign.

Personal dividend income
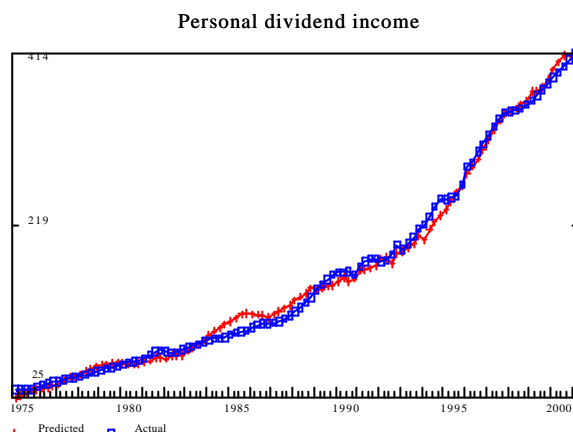
```
title Personal dividend income
# prfat -- Profits after tax
f prfat = niprf - nictax
# prfat is economic profits after taxes
f ub1div = @cum(ub1div,1.,.10)
f sprf = @cum(cprf,prfat,.10)/ub1div
:                         Personal dividend income
  SEE   =        8.34 RSQ   = 0.9945 RHO =   0.89 Obser  =  105 from 1975.100
  SEE+1 =        3.88 RBSQ  = 0.9943 DW  =   0.22 DoFree =  100 to   2001.100
  MAPE  =        6.20
    Variable name          Reg-Coef  Mexval  Elas   NorRes    Mean    Beta
  0 npdivi                - - - - - - - - - - - - - - - - -   163.32 - - -
  1 intercept             -22.97457    72.8  -0.14   181.52     1.00
  2 prfat                   0.20000     8.0   0.36     3.36   292.83  0.313
  3 prfat[1]                0.03237     0.1   0.06     2.76   287.54  0.050
  4 prfat[2]                0.04201     0.3   0.07     2.35   281.91  0.064
  5 sprf[3]                 0.45743    53.4   0.65     1.00   233.00  0.574
```

Government budget

The basic accounting of federal government expenditures in the NIPA may be summarized in the following table. The state and local account is similar except that the grants-in-aid item, *gfegia*, is a receipt rather than an expenditure.

```
+ gfr  Receipts
       gfrptx Personal tax and nontax receipts
       gfrprf Corporate profits tax accruals
```

```
        gfribt Indirect business tax and nontax accruals (Excises, duties, licenses,
               and (for State and Local) retail sales taxes)
        grfcsi Contributions for social insurance
- Current expenditures
        gfece  Consumption expenditures (including capital consumption)
        gfetp  Transfer payments (net)
        gfegia Grants-in-aid to State and local governments
        gfenip Net interest paid
        gfesls Subsidies less current surplus of government enterprises
        gfeald Less: Wage accruals less disbursements
= gfsurp       Current surplus (+) or deficit (-) in NIPA
+ gfconfc      Consumption of fixed capital
+ gfctr        Capital transfers received (Estate and gift taxes)
- gfv          Gross investment
- gfpnpa       Net purchases of non-produced assets
= gfenet       Net lending (+) or borrowing (-)
```

In Quest, the **Personal taxes and non-tax payments** are calculated by behavioral ratios (*pitfBR* and *pitsBR* for federal and state-and-local cases, respectively) relative to a specially created variable called *pTaxBase* defined as

      + Personal income
      + 0.5*Contributions to social insurance
      + Indirect business taxes.
      - Government transfer payments to persons

Half of Contributions to social insurance are added because in the federal most state income taxes, one is taxed on income *inclusive* of the employee's share of the Social security tax, but these contributions have been subtracted from Personal income in the NIPA. We also add into the tax base Indirect business taxes, such as the retail sales tax, for we are certainly taxed on the income with which the taxes are paid. Finally, we have subtracted Government transfer payments to persons on the grounds that most of these payments are either explicitly non-taxable or go to people with low incomes and are taxed at low rates.

The **Corporate profits taxes** are calculated by behavioral ratios (*gfrprfBR* and *gsrprfBR*) relative to Corporate profits. **Indirect business taxes**, in the federal case, are mostly alcohol, tobacco, and gasoline taxes, so they are modeled by a behavioral ratio (*gfribtBR*) relative to Personal consumption expenditure. In the state-and-local case, they also include retail sales taxes and franchise and licensing taxes. This broader base led to taking GDP as the base of the behavioral ration (*gsribtBR*). Finally, **Contributions for social insurance** are modeled by behavioral ratios (*gfrcsiBR* and *gsrcsiBR*) relative to earned income, approximated by National income less Net interest and Corporate profits.

Turning to the expenditure side, the GDP component, Government purchases of goods and services, is specified exogenously in real terms in three parts, federal defense (*gfdR*), federal non-defense (*gfnR*) and state and local (*gsR*). In addition, we specify exogenously in real terms government investment (*gfvR* and *gsvR*). **Current consumption expenditures** are then calculated by the identities

$$gfece \quad = \quad gfd + gfn - gfv$$
$$gsece \quad = \quad gs - gsv$$

Transfer payments, at the federal level, are divided among Unemployment insurance benefits, Transfers to foreigners, and Other. Unemployment insurance benefits are singled out for special treatment to get their automatic stabilizer effect. A behavioral ratio (*pituibBR*) makes them proportional to unemployment in real terms. The other two transfer payments are exogenous in real terms through the exogenous variables *gfetpfR* and *ogfetpR*. The last is, of course, the huge one. **Grants-in-aid**, *gfefiaR,* is also exogenous in real terms.

Both the federal government and the state and local governments both borrow and lend money. Consequently, they have both interest payments and receipts. The difference between the two levels of government, however, is profound; and the approach which works well for the federal government does not work at all for the state and local governments. For the **Net interest paid by the federal government**, which is a huge net borrower, we can calculate the overall deficit or surplus in each quarter and cumulate this amount to obtain a rough estimate of the net amount on which the government is earning or paying interest. By use of G's *fdates* command, we make the cumulation of the deficit or surplus begin at the same time that the regression begins. (The *fdates* command controls the dates over which the *f* commands work.) Because not all debt is refinanced instantly with the change in the interest rate, we use an exponentially weighted moved average of the rates, *frtb* or *srtb*, to multiply by the debt. We should then have

gfenip = InitialDebt*frtb + fcumdef*frtb

where *fcumdef* is the cumulated deficit of the federal government. The InitialDebt thus becomes a parameter in the regression equation. Notice that there is no constant term in this equation. We have therefore forced G to omit the constant term by placing a ! after the = sign in the *r* command. We have also included *rtb* as a separate variable in addition to *frtb* so that the regression can take an average of them to produce the best fit.

The same approach will not work at all for the **Net interest paid by state and local governments**, largely because these governments can borrow at low rates because the interest they pay is exempt from federal income tax. Thus, the rate they pay on their debt is far below the rate they receive on their assets, so the net indebtedness is not sufficient to make even a rough guess of the interest payments. Indeed, over the last twenty years the net indebtedness has grown while the net interest paid has become more and more negative. (The increase in the indebtedness is not immediately apparent from the NIPA, which show a positive surplus, *gssurp* in our bank.. The problem is that this surplus is not reckoned with total purchases of goods and services, *gs*, but only with consumption expenditures, *gsece*. The difference is that *gs* includes capital outlays while *gsece* excludes capital outlays but includes imputed capital consumption allowances. The cumulated surplus relevant for our purposes would be calculated with total expenditures, *gs*, and that surplus is negative throughout most of the last twenty years.)

In this situation, we have had recourse to a simpler device and assumed that state and local governments have tried to maintain both financial assets and liabilities roughly proportional to total purchases of goods and services, *gs*. Under that assumption, net interest payments should depend on *gs* and on its product with the interest rate. The fit is satisfactory and the elasticity of interest receipts with respect to *gs* just a little above 1.

gfenip -- Net Interest Paid by the Federal Government
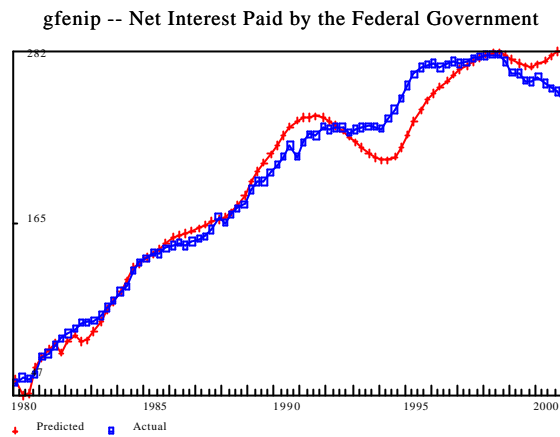

```
title gfenip -- Net Interest Paid by the Federal Government
f ub100 = @cum(ub100,1.,.1)
f frtb = @cum(cfrtb,.01*rtb,.1)/ub100
" calculate federal government deficit
fdates 1979.4 2005.4
f fcumdef = @cum(fcumdef,.25*gfenet,0.0)
fdates 1960.1 2005.4
f frXfcumdef = frtb*fcumdef[1]
f rXfcumdef = rtb*fcumdef[1]
r gfenip =  ! frtb,rtb, frXfcumdef, rXfcumdef
:           gfenip -- Net Interest Paid by the Federal Government
  SEE    =      11.79 RSQ  = 0.9710 RHO =   0.92 Obser  =    85 from 1980.100
  SEE+1  =       4.54 RBSQ = 0.9699 DW  =   0.15 DoFree =    81 to   2001.100
  MAPE   =       5.40
    Variable name         Reg-Coef  Mexval  Elas   NorRes     Mean    Beta
  0 gfenip              - - - - - - - - - - - - - - - -    192.94  - - -
  1 frtb                 115.71715     0.7   0.04   101.02     0.07
  2 rtb                    3.51570     7.1   0.12    96.61     6.76   0.144
  3 frXfcumdef            -1.64228   132.5   0.87     1.01  -102.48  -1.215
  4 rXfcumdef              0.00082     0.6  -0.04     1.00 -9513.08   0.065
```

Net Interest Paid by State and Local Governments

Predicted   Actual
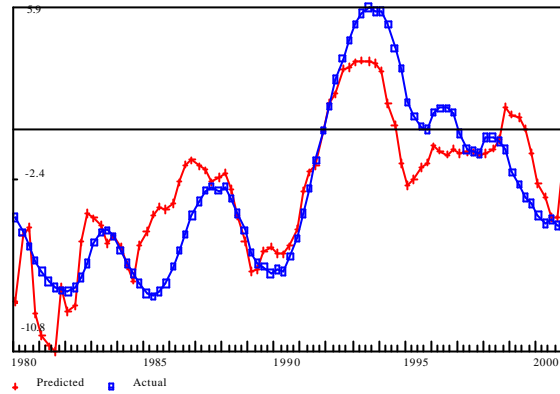
```
title Net Interest Paid by State and Local Governments
f gsXrtb = gs*rtb
r gsenip =   gs, gsXrtb
:             Net Interest Paid by State and Local Governments
  SEE   =       2.09 RSQ   = 0.7190 RHO =   0.87 Obser  =   85 from 1980.100
  SEE+1 =       1.14 RBSQ  = 0.7121 DW  =   0.27 DoFree =   82 to   2001.100
  MAPE  =      83.93
    Variable name          Reg-Coef  Mexval  Elas   NorRes    Mean    Beta
  0 gsenip                - - - - - - - - - - - - - - - - -   -2.97  - - -
  1 intercept              -0.99754    0.7   0.34     3.56     1.00
  2 gs                      0.01387   77.0  -3.18     2.57   680.14   0.874
  3 gsXrtb                 -0.00278   60.4   3.84     1.00  4105.95  -0.751
```

Subsidies less current surplus of government enterprises are small and have been taken
exogenously in real terms for all levels of government. **Wage accruals less disbursements** are
generally zero and have been left exogenous in nominal terms.

With these items, we are able to calculate the **Current surplus (+) or deficit (-)** on the NIPA
basis. To calculate **Net lending (+) or borrowing (-)**, however, we need a few more items. The
most important of these is **consumption of fixed capital.**

Until fairly recently, all government purchases were considered current expenditures in the NIPA.
Thus, the construction of a road entered into the GDP only in the year it was built; services from the
road were not counted as part of the GDP. In the private sector, however, the consumption of
fixed capital, depreciation expense, enters into the price of goods consumed. Thus, a capital
expenditure in the private sector is counted in GDP twice, once as fixed investment in the year in
which it is made and then again in the prices of goods and services as it is consumed in future years.
(In Net Domestic Product, this second appearance has been removed.)   To give government
capital formation similar treatment, the NIPA have recently begun to distinguish between current
expenditures and capital expenditures. The capital expenditures are then amortized to create a
consumption of fixed capital expense. Our technique for estimating this consumption given
previous investment is similar to what we used in the private sector. Here are the equations for the
two level of governments.

FederalConsumption of Fixed Capital
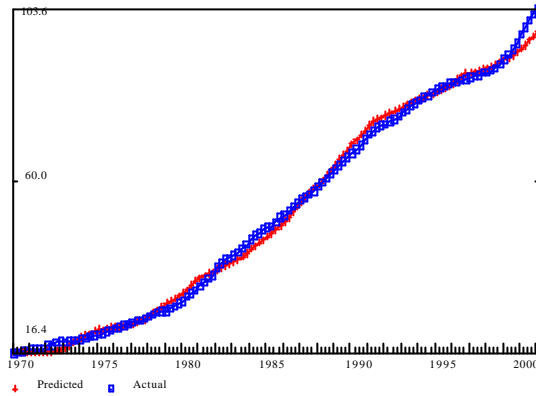
```
ti FederalConsumption of Fixed Capital
fex gfvR = gfv/gdpD
f gfv = gfvR*gdpD
f ub02 = @cum(ub02,1.,.02)
f gfvrep = gdpD*@cum(gfvstk,gfvR,.02)/ub02
:                   FederalConsumption of Fixed Capital
  SEE   =       1.71 RSQ  = 0.9960 RHO =   0.97 Obser  =   125 from 1970.100
  SEE+1 =       0.45 RBSQ = 0.9960 DW  =   0.07 DoFree =   123 to   2001.100
  MAPE  =       3.05
    Variable name        Reg-Coef  Mexval  Elas   NorRes      Mean    Beta
  0 gfconfc              - - - - - - - - - - - - - - - -     52.27  - - -
  1 intercept            -4.18613    45.4  -0.08   250.03      1.00
  2 gfvrep                1.07878  1481.2   1.08     1.00     52.34   0.998
```


State and Local Consumption of Fixed Capital
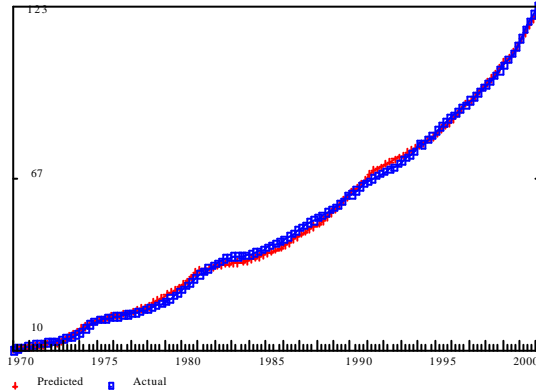
```
ti State and Local Consumption of Fixed Capital
fex gsvR = gsv/gdpD
f gsv = gsvR*gdpD
f ub04 = @cum(ub04,1.,.04)
f gsvrep = gdpD*@cum(gsvstk,gsvR,.04)/ub04
:                   State and Local Consumption of Fixed Capital
  SEE   =       1.34 RSQ  = 0.9981 RHO =   0.96 Obser  =   125 from 1970.100
  SEE+1 =       0.38 RBSQ = 0.9981 DW  =   0.08 DoFree =   123 to   2001.100
  MAPE  =       2.76
    Variable name        Reg-Coef  Mexval  Elas   NorRes      Mean    Beta
  0 gsconfc              - - - - - - - - - - - - - - - -     51.39  - - -
  1 intercept            -8.01786   193.3  -0.16   530.19      1.00
  2 gsvrep                0.69125  2202.6   1.16     1.00     85.94   0.999
```

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The spill rates were chosen after some experimentation to get a good fit. The replacement calculated for the federal government is fairly close to the NIPA capital consumption series; for state and local government, however, the calculated replacement is much above that used in the NIPA.

In the new accounting **Estate and gift taxes** are no longer counted as government revenues but appear, more correctly, as **Capital transfers.** They have been treated by behavioral ratios (*gfctrBR* and *gsctrBR*) to Personal income on the presumption that increases in income also increase estates and gifts.

The final item in the government accounts is the **Purchases of non-produced assets** such as land or stocks. These purchases cannot go into GDP, precisely because the land or the stock is not produced. On the other hand, they enter the cash "bottom line" of the governments. They are taken as exogenous in real terms with the variables *gfpnpaR* and *gspnpaR*.

From these variables, the complete government accounts as set out at the beginning of this section can be computed.

The Stock Market Value

We have used the **real stock market value** variable, *sp500R*, has been used in a number of equations. Now we turn to trying to explain the variable with other variables in the model. Fundamentally, the value of a stock should be present value of the stream of future profits discounted by the rate of interest. If we put the profits in real terms, then the interest rate used should be a real rate. Basically, our equation for *sp500R* relates it to the present value of future profits by presuming that both profits and interest rates are expected to remain at their present level in real terms. Both profits and interest rates have been exponentially smoothed to reduce variability that was not reflected in the stock market series. Profits are likely to be discounted at rates considerable above the Treasury bill rate. After trying several values, we settled on adding 5 percentage points to the "perceived" Treasury bill rate. The regression coefficient on this variable was then constrained to give it an elasticity of 1. A time trend was also allowed.

The results below show this equation estimated only through 1994.4, roughly the beginning of the present bull market. Notice that the 1987 "correction" brought the market back close to the value calculated by this equation. The lines to the right of the vertical line compare the actual values of the stock market variable with the values which would be "justified" by the equation estimated over the previous fifteen years. The time trend fortunately turns out to be small, a quarter of a percent per year of the mean value of the index.

S&P 500 Index
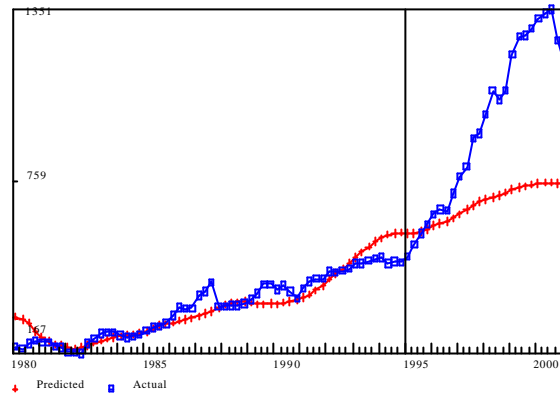
```
ti S&P 500 Index
f ub10 = @cum(ub10,1.,.1)
f rtbexs = @cum(crtbex,5.+rtbex,.10)/ub10
f niprfs = @cum(cniprf,niprf,.10)/ub10
fex sp500R = sp500/gdpD
f DiscProfit = (niprfs/rtbexs)/gdpD
# constrain to give Discounted Profits an elasticity of 1.
con 1000 7 = a2
:                              S&P 500 Index
  SEE   =      45.19 RSQ   = 0.8127 RHO =   0.90 Obser  =   60 from 1980.100
  SEE+1 =      23.47 RBSQ  = 0.8061 DW  =   0.19 DoFree =   57 to   1994.400
  MAPE  =      10.41 Test period:   SEE   351.54 MAPE   25.66 end  2001.100
    Variable name          Reg-Coef  Mexval  Elas   NorRes    Mean    Beta
  0 sp500R               - - - - - - - - - - - - - - - -   325.90  - - -
  1 intercept            -253.72411    31.5  -0.78   28.32     1.00
  2 DiscProfit              6.72921   384.4   1.04    1.62    50.30   0.758
  3 time                    8.73035    27.4   0.74    1.00    27.62   0.362

id sp500 = sp500R*gdpD
```

The Exogenous Variables

To facilitate the use of the model, here is a list of all the exogenous variables in one place.

lfc	Civilian labor force
pop	Population
gm1	Growth rate of M1
fgndem	Foreign demand, used in export equation
relpri	Prices of imports relative to prices of exports, used in inflation equation
fefaci	Exports of factor income
fifaci	Imports of factor income
taxacts	Dummy for tax acts affecting construction
d80	Dummy in the export equation
disaster	Dummy for hurricane and earthquake in capital consumption
nbtrpBR	Behavioral ratio for business transfer payments
nbtrppBR	Behavioral ratio for business transfer payments to persons
nsd	Statistical discrepancy

nwald   Wage accruals less disbursements

In the government sector, there are usually parallel variables for federal (in the first column below) and state-and-local governments (in the second column). All variables ending in *R* are in constant prices. Those ending in BR are ratios to some other variable as explained in the government section above.

Federal	S&L	Description
gfdR		Purchases of goods and services for defense
gfnR	gsR	Purchases of goods and services, non-defense
gfvR	gsvR	Capital investment
pituibBR		Unemployment insurance benefit rate
gfetpfR		Transfer payments to foreigners
ogfetpR	gsetpR	Other transfer payments
gfeifBR		Interest payments to foreigners
pitfBR	pitsBR	Personal tax rates
gfribtBR	gsribtBR	Indirect business tax rate
gfrprfBR	gsrprfBR	Profit tax rates
gfrcsiBR	gsrcsiBR	Social security tax rates
gfctrBR	gsctrBR	Estate and gift tax rates
gfpnaBR	gspnaBR	Ratio for purchases of non-produced assets
gfeslsR	gseslsR	Subsidies less surplus of government enterprises
gfetpfR		Transfer payments to foreigners
gfegiaR		Federal grants in aid to state and local government
gfeald	gseald	Wage accruals less disbursements

3. Historical Simulations

In the following graphs, the heavy line with no marking (blue, if you are reading this in color) is the actual historical course of the variable. The (red) line marked with +'s is the simulation with the stock market variable at its historical values, while the (green) line marked with x's is the simulation using the equation for the stock market equation.

u -- Unemployment
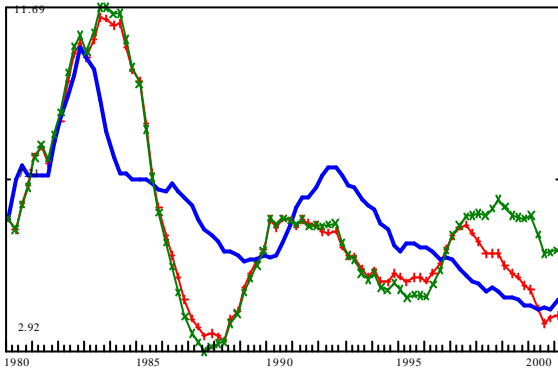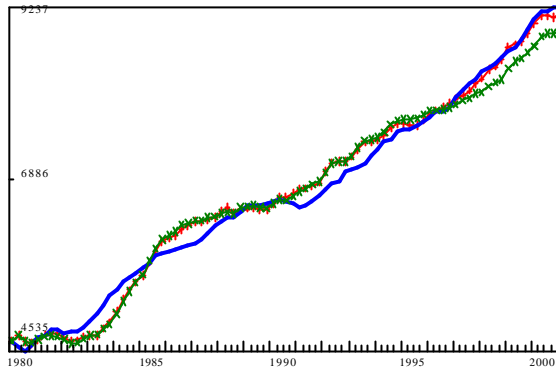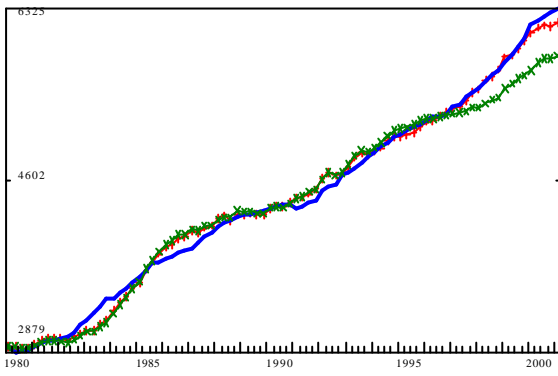
b.u   c.u   d.u

gdpR -- Real Gross Domestic Product

b.gdpR   c.gdpR   d.gdpR

cR -- Personal Consumption Expenditure

b.cR   c.cR   d.cR

vfnreR -- Equipment Investment

b.vfnreR   c.vfnreR   d.vfnreR

vfnrsR -- Non-residential Structures

b.vfnrsR   c.vfnrsR   d.vfnrsR

vfrR -- Residential Construction

b.vfrR   c.vfrR   d.vfrR

feR -- Exports

b.feR   c.feR   d.feR

fiR -- Imports

b.fiR   c.fiR   d.fiR
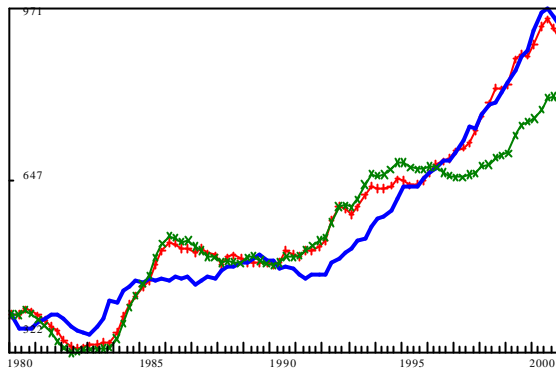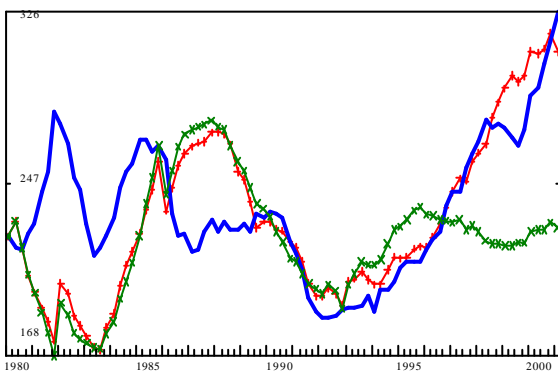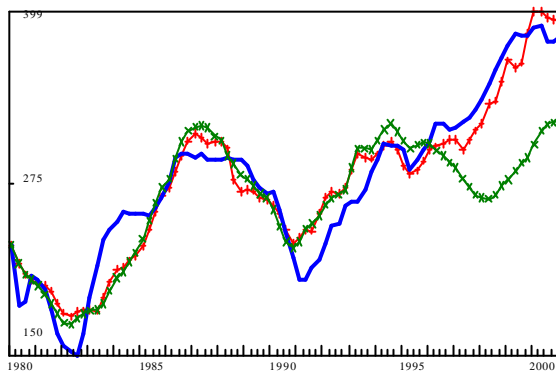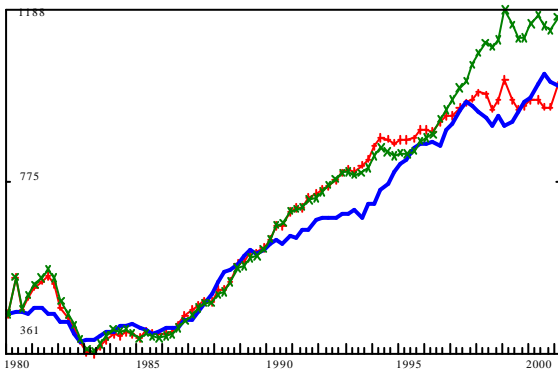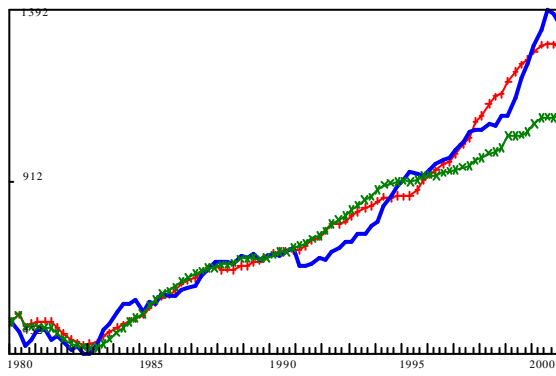
gdpD -- GDP Deflator
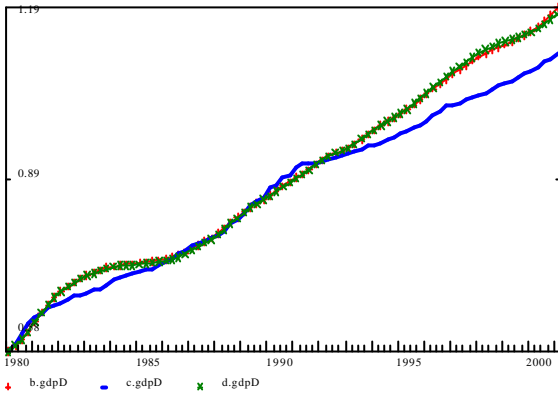
infl -- Inflation
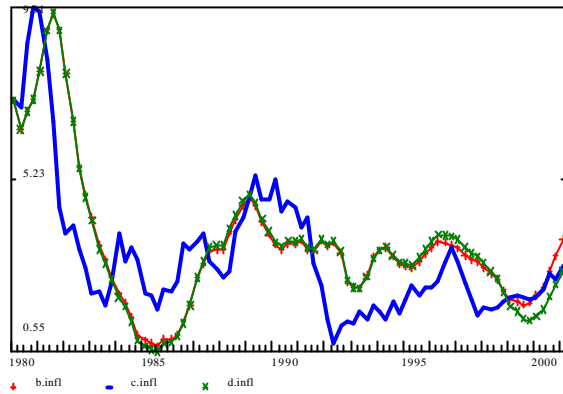
pisav -- Personal Saving
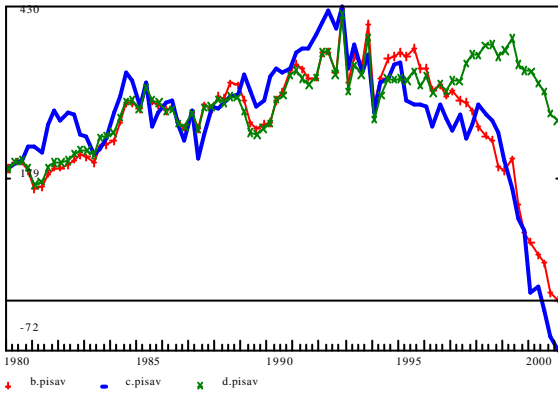
sp500R -- S&P 500 Stock Market Index, Real
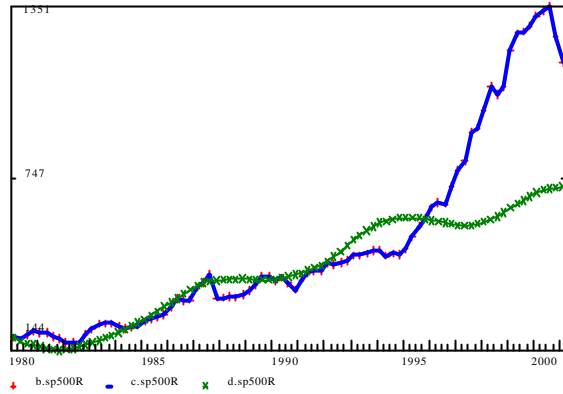
rtb -- Rate on New 90-day Treasury Bills

v1 -- M1 Velocity

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b.gfenet   c.gfenet   d.gfenet   b.gsenet   c.gsenet   d.gsenet
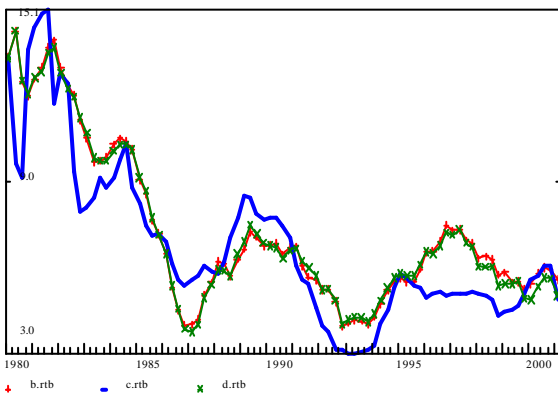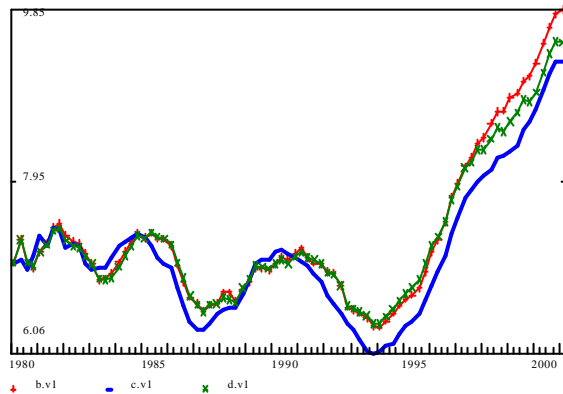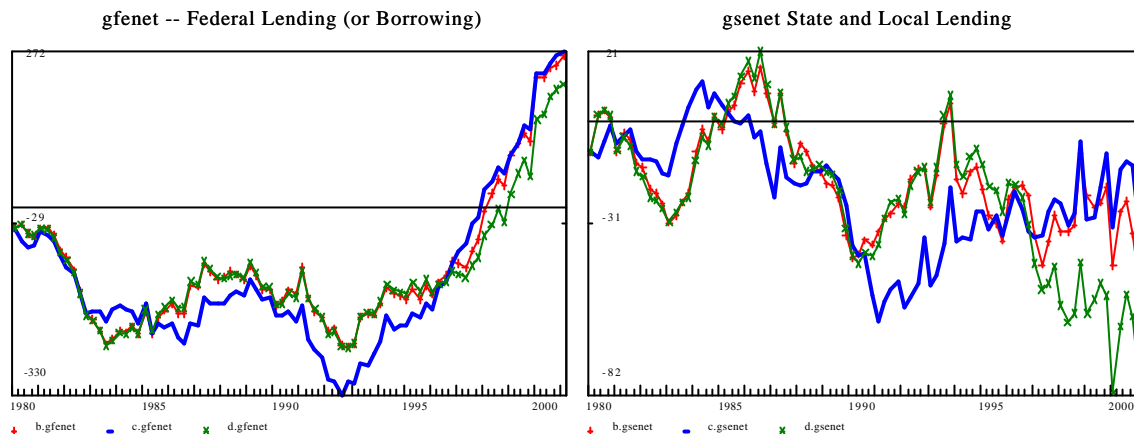
Up through 1992, the stock market equation worked well and there is essentially no difference between the two simulations. Both track the real variables, such as GDP, Personal consumption expenditure, residential construction, equipment investment, and employment fairly well with the exception that the model produces a stronger boom in 1986 and 1987 than actually happened. After 1996, the story is quite different. The simulation with the stock market taking its normal course as an endogenous variable shows, to be sure, a steady, moderate growth in the stock market but a significant recession in 1996-1997 followed by a weak recovery with a rising unemployment rate that almost reached the levels of 1981-1982 in 1999 before a slight recovery in 2000. In sharp contrast, the simulation with the stock market variable set exogenously to its actual, historical values gave fairly close simulations of the real variables up through 2000. In particular, the personal savings rate falls and Personal consumption expenditures rise in this simulation very much as they actually did historically.

The story is a little different for the price level. The simulations track it quite well up to about 1990; thereafter it gets above the historical values and stays there to the end of the period. In other words, the inflation rate misses on the high side for a year or so and then remains very close to the actual. In theory, tight money (indicated by a high monetary velocity) should have reigned in the economy by reducing investment and consumption. The M1 velocity graph, however, shows that the differences of the simulation from the historical velocity were small in comparison with the changes which were taking place historically in the velocity. It was therefore difficult to find a measure of monetary tightness which would show up as statistically useful in estimating the equations.

The conclusions I draw from these results are:
> The stock market is quite important to the economy.
> Given the stock market behavior, the model can predict the rest of the economy, especially its real variables, fairly well.
> The boom in the stock market which began in 1995 is responsible for the strong economy of the period 1996 - 2000.
> This the causes of this boom in the market lay outside the U. S. economy.

These external causes are not hard to find. Beginning in 1996, weakness in Asian and other economies led to an influx of foreign investment into the U.S. stock market. Without the externally
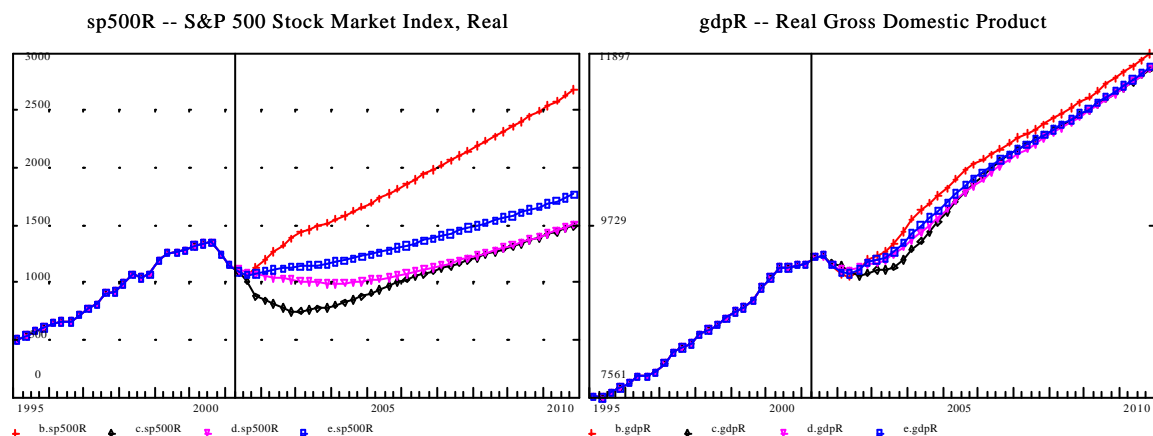
driven rise in the stock market, the years 1996 - 2000 would have shown weak but positive growth. The exceptional prosperity of the period was the result of the bull market superimposed on a fundamentally stable but not especially dynamic economy.
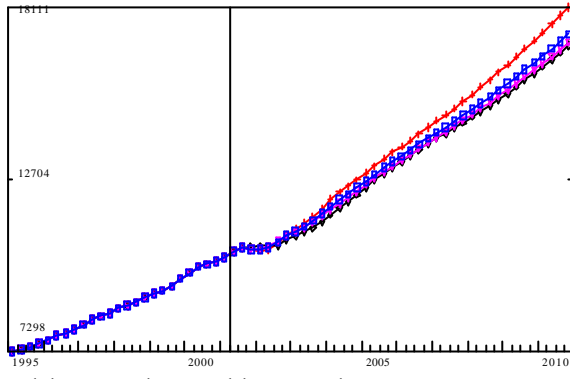
4. Alternative Forecasts

To study the effect of the stock market on the cyclical evolution of the American economy in the coming years, we have formulated four alternative projections. They differ only in the projection of the real value of the stock market index, *sp500R*. All four alternative projections are made by adding a factor to the endogenous equation for *sp500R*. In naming the alternatives, we expand on the custom of distinguishing between "bulls" and "bears". The alternatives are:

Name	Mark	Description
Bull	+ plus	The add factor reaches a minimum in 2001.3, climbs back to its highest historical value by 2002.4, and continues to grow at the same rate at which it grew from 1996 to the peak in 2000. (Red, if you are reading on a screen.)
Sheep	? square	The add factor stays where it is likely to be in 2001.3. (Blue)
Bear	? triangle	The add factor is generated automatically by the rho-adjustment process. (Purple)
Wolf	? diamond	The add factor, which was 400 in 2001.1 hits 100 by 2001.4 and drops on down to -100 by 2001.4, where it stays for a year before moving up to -10 by the end of 2005. (Black)

All of the alternatives reflect the Bush tax cut of 2001 and otherwise use middle-of-the-road projections of exogenous variables. Here are the comparison graphs.
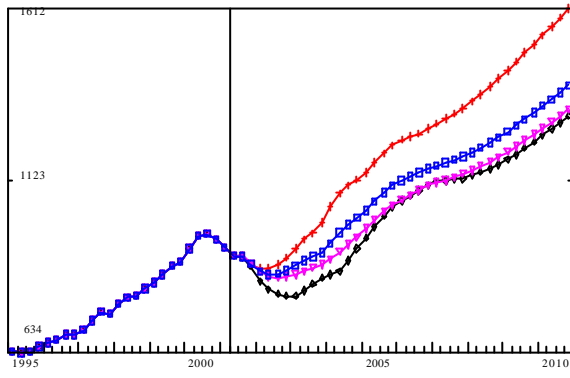
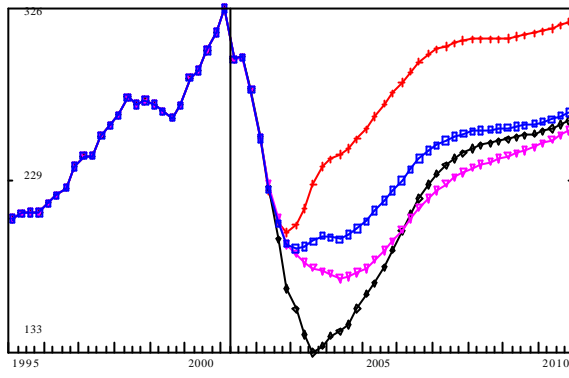

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gdp -- Gross Domestic Product
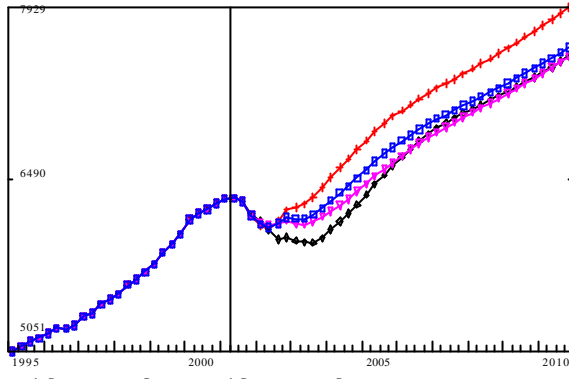
b.gdp   c.gdp   d.gdp   e.gdp

cR -- Personal Consumption Expenditure

b.cR   c.cR   d.cR   e.cR

vfnreR -- Equipment Investment

b.vfnreR   c.vfnreR   d.vfnreR   e.vfnreR

vfnrsR -- Non-residential Structures

b.vfnrsR   c.vfnrsR   d.vfnrsR   e.vfnrsR

vfrR -- Residential Construction

b.vfrR   c.vfrR   d.vfrR   e.vfrR

feR -- Exports

b.feR   c.feR   d.feR   e.feR

fiR -- Imports

b.fiR   c.fiR   d.fiR   e.fiR

u -- Unemployment

b.u   c.u   d.u   e.u

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infl -- Inflation | v1 -- M1 Velocity
rtb -- Rate on New 90-day Treasury Bills | pisav -- Personal Saving
gfenet -- Federal Lending (or Borrowing) | gsenet State and Local Lending
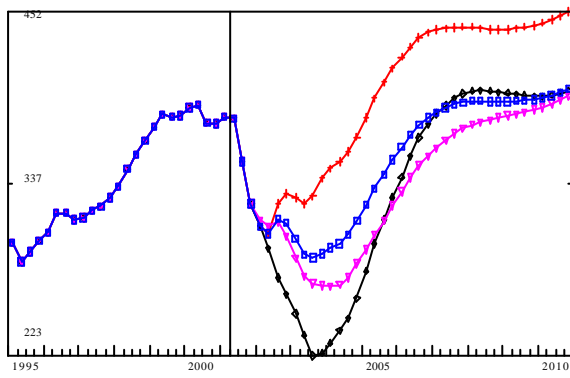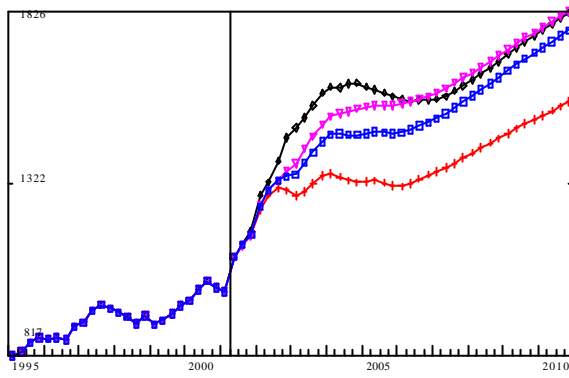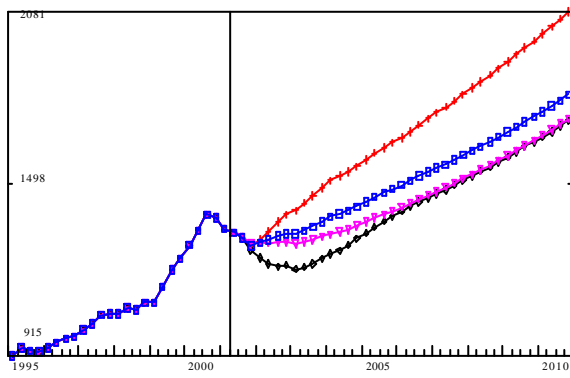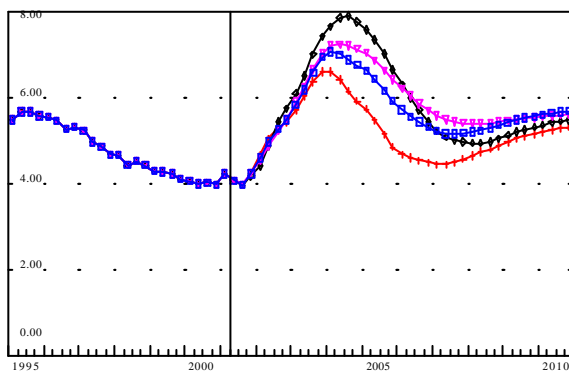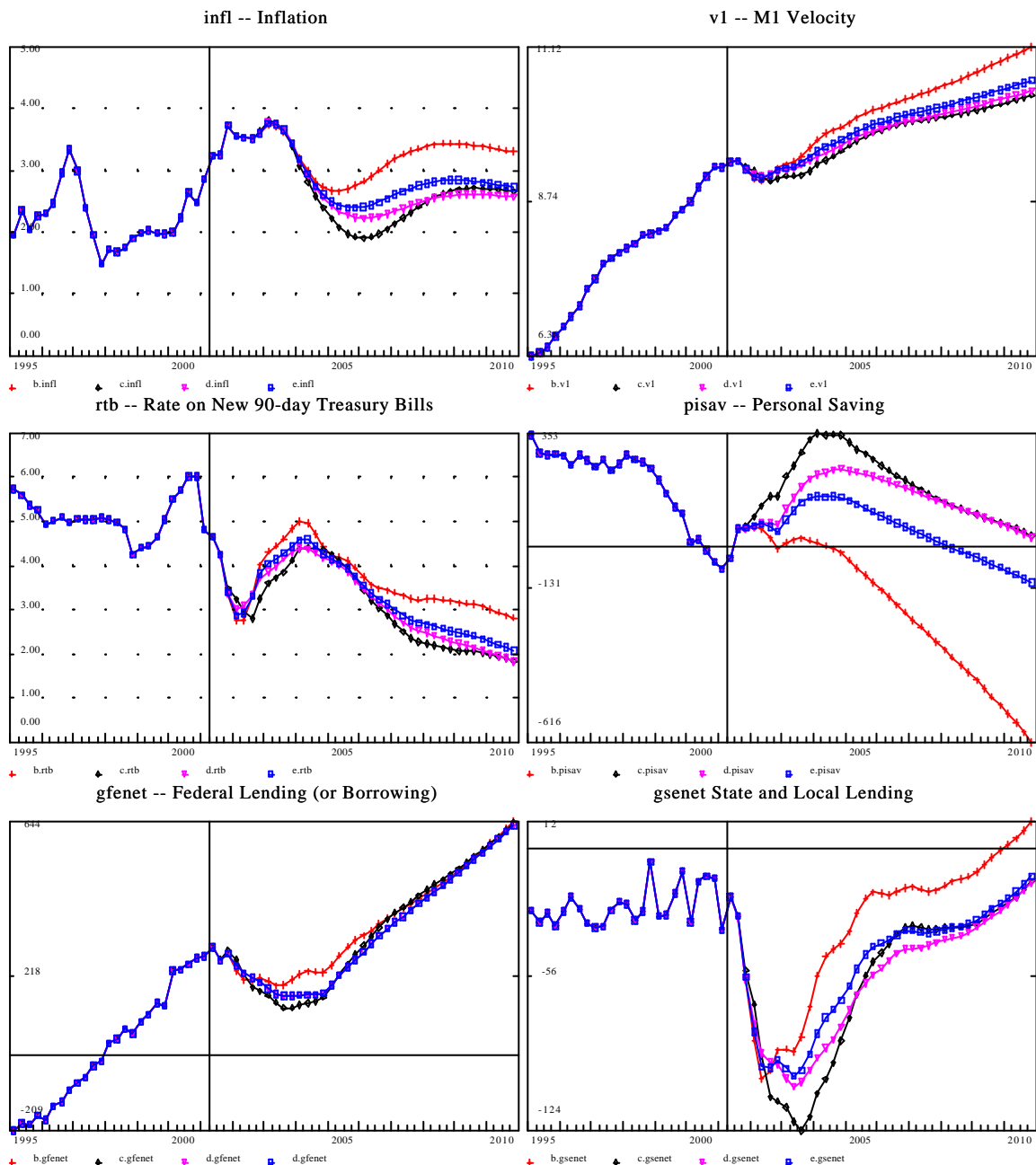
All the alternatives agree that we are in for a considerable recession beginning in the last quarter of 2001. For comparison, it is useful to remember that the recession beginning in 1990 lasted three quarters and saw a drop of 1.7 percent in real GDP. The one just ahead should also last three quarters (or four for the Wolf scenario) but the drop in real GDP may be on the order of 2 or 3 percent. Looking over the graphs above show much greater drops in consumption and investment. Exports and imports, however, act as very strong stabilizers, and -- in this model -- respond very quickly to changes in the stock market. The response is so fast that Bull, which activates the export-import stabilizers least of the four, turns out to have the sharpest and deepest recession, at 3 percent in three quarters. Wolf, which activates them most, has a 2.9 percent drop over four quarters, while Sheep loses 2.6 percent over three quarters and Bear drops only 2.0 percent over three quarters. The details of this short-run response can be seen clearly in the following graph.

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pgppR -- Peak Real Gross Private Product
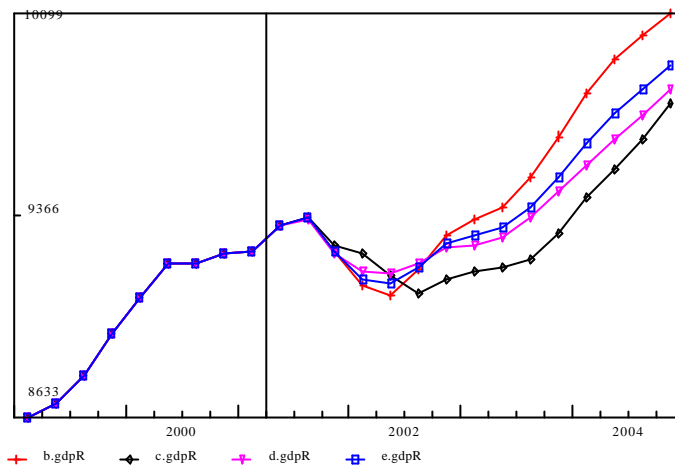
Once the recovery is underway, the alternatives assume the expected order according to speed of Bull, Sheep, Bear, and Wolf. The maximum difference between Wolf and Bull is 4.2 percent.

The combination of the tax cuts and the recession will wipe out half of the Federal budget surplus. The model does not distinguish between the social insurance trust funds and the general budget, but it is clear that the general budget will be in deficit during the recession. The State and Local deficit is sharply increased by the recession, and one can expect cut backs in expenditures to avoid these deficits.

After the recession, unemployment stabilizes at about 5.5 percent and inflation at about 2.5 percent. Personal savings, after rising during the recession, shows a disturbing tendency to diminish.

All in all, it appears that the model is capable not only of generating a substantial cycle but also, when the exogenous variable are stable, of producing stable growth at plausible levels of unemployment and inflation.

Optimization in Models

Up to this point, we have estimated equations in isolation and then combined them into a model and observed how the model worked. Occasionally, we have revised the estimate of some regression coefficient to improve the functioning of the model. In this chapter, we will see how to modify coefficients in a comprehensive way to improve the performance of the model in historical simulation. The same techniques, with a different objective function and different parameters, can then be used to design policies. Let us begin, however, with improving the performance of the model in historical simulation.

1. Improving the historical simulation

Creating an Objective Function

The first step in optimizing must be to create an objective function. This objective function must be built into our model. Our software uses the convention that it **minimizes** the value in the last period of the simulation of a variable which must have the name *objective*. For example, to optimize the performance of the Quest model in historical simulation, we would probably initially want to concentrate on real GDP (*gdpR*) and the GDP deflator (*gdpD*). Let us say that we want to minimize the sum of the squares of their relative, fractional differences from their historical values. We then need to record the historical values in variables which will not be changed in the model, so we create two exogenous variables, *gdpRX* and *gdpDX* for that purpose by the equations:

        fex gdpRX = gdpR
        fex gdpDX = gdpD

The relative difference between the model's real GDP in any period and the historical value for that period would be (gdpR-gdpRX)/gdpRX and for the GDP deflator it would be (gdpD-gdpDX)/gdpDX. The contribution to the objective function from these discrepancies in any one period would be

        f obj = @sq((gdpR-gdpRX)/gdpRX)+@sq((gdpD-gdpDX)/gdpDX)

where *@sq( )* is the squaring function. Finally, the objective function itself, the sum over all periods of these period-by-period contributions, would be the value in the last period of the simulation of the variable *objective* defined by

        f objective = @cum(objective, obj, 0.)   .

These statements can be conveniently placed at the end of the Master file of the model just before the "check" commands.

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Selecting parameters to vary

With the objective function in place, the next step is to select from all the regression coefficients s in the model those which will be varied in looking for an optimum. One might ask, "Why not vary all of them?" Our objective function, however, is quite a complicated function of all these coefficients, so the only feasible optimization techniques are those that involve some sort of trial-and-error search with the whole model being run to evaluate the objective function for each proposed point, that is, for each set of regression coefficient values. The number of points that has to be searched increases with the dimension of the point. We will see, however, that optimizing with respect to a relatively small number of coefficients – a dozen or so – can produce a substantial improvement in the Quest model.

The optimization method we will use is known as the simplex method . A *simplex* in n-dimensional space is a set of n+1 points in that space. For example, a triangle is a simplex in 2-dimensional space and a tetrahedron is a simplex in 3-dimensional space. The method requires that we specify an initial simplex of points; it will then take over, generate a new point, and, if that point is better than the old worst point in the simplex, drop the worst point and add the new point to the simplex. It has four different ways of generating new points. First it *reflects* the worst point through the midpoint of the other points. If that works, it tries to *expand* by taking another step of the same size in the same direction. If the expansion gives a better point than did the reflection, that point is added to the simplex and the worst point is dropped. If the reflection gave a point better than the worst point but the expansion did not improve on it, the reflected point is added to the simplex and the worst point dropped. If the reflection failed to give a point better than the worst point, the algorithm *contracts*, that is, it tries a point halfway between the worst point and the midpoint of the other points. If this point is better than the worst point, it is added to the simplex and that worst point dropped. Finally, if all of these trials have failed to yield a point better than the worst point, the algorithm *shrinks* the simplex towards the best point by moving all the other points halfway towards it. When the value of the objective function is practically the same at all the points and the points are close together, it stops.

Our task is to supply the initial simplex. One obvious point for inclusion is the values of the coefficients estimated by the original regressions. We specify the other points by varying each coefficient, one-by-one, from this base. For each coefficient, we will specify a "step size" for this variation. The initial points of the simplex are then the original values of the parameters that may be varied and then, for each parameter, a point with that parameter increased by its "step size" and all the other parameters at their original values. Note that with n parameters, this method will give n+1 points, a simplex in n-dimensional space.

Mechanically, how do we specify the parameters to be varied and their step sizes? An example for Quest will be helpful. We will optimize on parameters from the consumption function, that is, the equation for *cRpc,* and the most important of the investment equations, that for *vfnreR.* For ease of reference, here are excepts from the regression results of the consumption equation.

```
:                    Personal Consumption per capita
    Variable name         Reg-Coef  Mexval   Elas   NorRes     Mean    Beta
  0 cRpc                 - - - - - - - - - - - - - - -   17419.94  - - -
  1 intercept             785.42866    1.1    0.05   792.49      1.00
  2 yRpc                    0.77579   31.8    0.86     9.50  19284.28  0.711
  3 dyRpc                  -0.39068    8.5   -0.00     9.25    104.29 -0.018
 ....
 13 piipcbRpc             -0.24243    0.1   -0.01     1.29    507.34 -0.008
 14 intsavRpc             -0.48752   10.1   -0.03     1.09   1151.80 -0.044
 15 rtbexXdi              -0.00161    1.5   -0.01     1.03  55178.74 -0.015
 16 ur                  -1417.2942    1.3   -0.01     1.00      0.17 -0.020
```

Examination of the historical simulations shown in the previous chapter shows that the equipment investment equation is a major generator of the boom in the mid 1980's that was much stronger in the historical simulation than in reality. Could inclusion of an unemployment variable in this equation help stabilize the model? One could argue that, in times of tight employment, capacity constraints may result in orders for capital goods may not be filled promptly so that actual investment may be less than would be desired on the basis of other factors. The number of persons unemployed, *ue*, was put in with the following results:

```
f ue = lfc -emp
:                        Equipment Investment
    Variable name        Reg-Coef  Mexval  Elas   NorRes     Mean    Beta
  0 vfnreR               - - - - - - - - - - - - - - - -    510.68  - - -
  1 intercept             36.82448    1.7    0.07   120.92     1.00
  2 repEq                  0.97140  465.9    0.77    10.00   404.92  0.629
  .....
 24 dsp500R[7]             0.02980    0.1    0.00     1.05     8.60  0.005
 25 ue                    -4.60458    2.4   -0.07     1.00     7.64 -0.036
```

The unemployment variable got a negative coefficient, which would only make the cycles worse. No doubt we have here a case of simultaneous equation bias, for booming investment will drive down unemployment. Rather than try instrumental variables or other simultaneous equations techniques, let us just make this coefficient one of the variables on which we optimize.

The specification of which parameters to use in optimization and their step sizes is now provided by the following file, which we may call OptSpec.opt.

```
20
vfnreR
#  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
   .1 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 .1
cRpc
#  1     2 3  4  5  6  7  8  9 10 11 12 13     14 15 16
   .1 .001 0  0  0  0  0  0  0  0  0  0  0   .005  0  1
```

The first line of the file specifies the maximum number of parameters which will be varied in the course of the optimization. It does not hurt if it is larger than the number actually used. Here we have set the maximum at 20 but will only use 6.

The next line says that some parameters will come from the equation for *vfnreR*. The third line begins with a # which marks it as simply a comment ignored by the program. For us, however, it is very useful since it numbers the 25 regression coefficients which occur in the equation for *vfnreR*. The line below it gives the step sizes for each of these 25 coefficients. A coefficient given a step size of 0 is not involved in the optimization. Thus we see that coefficient 1, the intercept, is given a step size of .1 and that the coefficient of *ue* is also given a step size of .1.

The next triplet of lines does the same for three coefficients in the *cRpc* equation, the intercept, the coefficient of the inflationary interest that "should" be saved, and the reciprocal of the unemployment rate.

Note that in both equations, the intercept is included among the variables on which we optimize. The reason is that, unless a variable happens to have a mean of zero, changing the coefficient on it will require a change in some other variable's coefficient to keep the sum of errors in the equation zero. The intercept is a natural choice for this other variable since it seldom has an economic significance which we want to preserve.

With this file created, we are ready to optimize our objective function.

Optimizing

When the model with the objective function has been built (by clicking Model | Build in G), we can run it in optimizing mode. Click Model | Run and then in the top right corner of the screen in the panel labeled "Type of Simulation" click the radio button for "Optimizing". Fill in the dates of the simulation and the "fix" file as usual. Specify the name of the bank which will contain the optimized model run. I usually call it "Optima", but any word of 8 or less letters and numbers will do. Finally, in the window labeled "Optimization file name", give the name of the file created in the previous step. In our case, it is OptSpec.opt, which is what the program puts in that window by default. The root-name of this file (the part before the .opt) will be used to label several of the files resulting from the optimization. Then click OK. You will then get a black DOS screen with the usual] prompt. You can provide a title for the run with a "ti" command or supplement the "fix" file. When running Quest over history, I often give the "skip sp500R" here to use historical values of the S&P 500 index. When you have no further fixes to add, give the command "run" as usual.

When optimizing, the model does not print dates and the values of variables being checked. Instead, it reports for each move of the simplex whether the action was to reflect, expand, contract, or shrink. It also shows the value of the objective function at the best and worst points of the simplex.

The implementation of the simplex method used by our program is borrowed from section 10.4 of *Numerical Recipes in C* by William H. Press *et al.* (Cambridge, 1988; the code and text is available on the Internet at www.nr.com .) This code seems prone to reach local minima. Therefore, when an optimum is reported by the borrowed code, our routine takes it as a starting point and then uses the step sizes to vary it. If one of the new points is better than the supposed
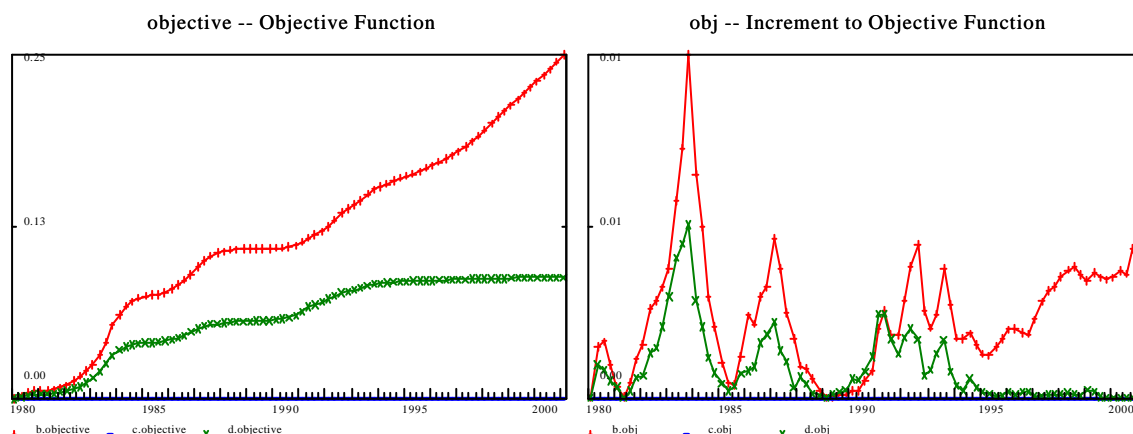
optimum, the algorithm is started again, with the message "Starting or restarting optimization" printed on the screen.
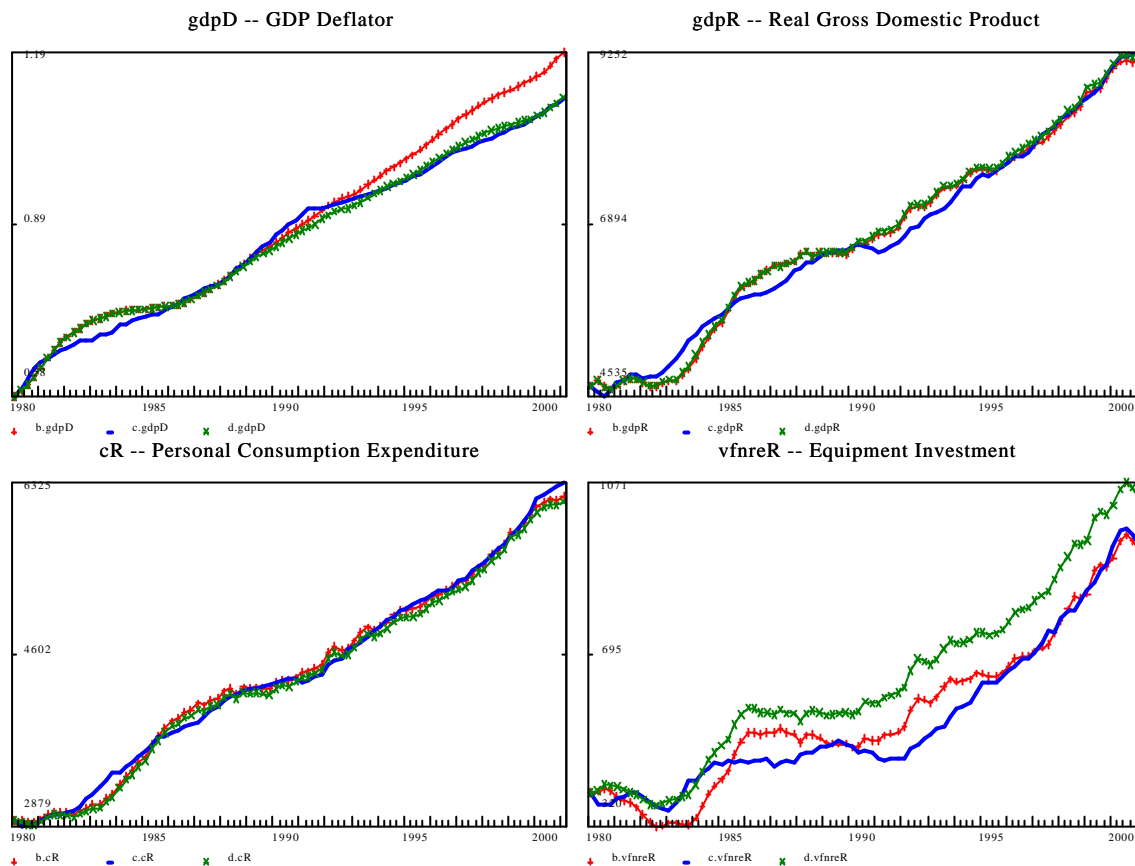
When no further improvement appears possible, you will get a list of the parameters with their starting values and their optimized values. This information will also be written into the file Changes.chg. When you then tap any key the model will be run with the optimized parameters and the results stored in the bank you indicated on the Run model screen.

When Quest was optimized with the objective function given above with respect to the parameters specified by the OptSpec.opt file shown above, the coefficients were changed as follows:

```
Resulting coeficients after maximization (183 runs).
Variable     Old:       New:
vfnreR
  intercept  36.8245     36.2423
  ue         -4.6046     -0.6041
cRpc
  intercept  785.4286    804.7291
  yRpc         0.7758      0.7669
  intsavRpc   -0.4875     -0.4898
  ur       -1416.2942  -1464.8260
```

One might suppose that these changes are so small that the optimization must have made little difference in the objective function. That impression, however, is quite misleading as shown in the graphs below. In them, the heavy (blue) line with no marking of points is the actual, historical line. (In the first two graphs, it lies along the horizontal axis, for of course the historical data fits itself perfectly.) The (red) line marked with + is generated by the model before optimization; the (green) line marked with x is from the optimized model. Remember that we are trying to minimize errors, so lower is better.

gdpD -- GDP Deflator   gdpR -- Real Gross Domestic Product

cR -- Personal Consumption Expenditure   vfnreR -- Equipment Investment

From the first graph, we see that the optimization achieved a 65 percent reduction in the objective function. The second graph shows that the contribution to the error fell essentially to zero over the last five years. I must confess that I was surprised by how much was achieved by such small changes in so few parameters. The second and third graphs show that the main improvement lay in the GDP deflator, while real GDP was little changed.

However, the last two graphs, especially the last, point to a problem. The simulation of equipment investment in the optimized model is terrible! In specifying our objective function, we implicitly hoped that if we had a good simulation for real GDP, we would have a good fit for its components. That hope, however, proved false. The lesson seems to be that if some parameters of the equation for a particular variable are included in the optimization, that variable needs to be in the objective function.

With that lesson in mind, we go back and respecify the objective function to include both equipment investment and personal consumption as follows:

```
fex gdpRX = gdpR
fex gdpDX = gdpD
fex vfnreRX = vfnreR
fex cRX = cR
f obj = @sq((gdpR-gdpRX)/gdpRX)+@sq((gdpD-gdpDX)/gdpDX) +
    0.1*@sq((vfnreR-vfnreRX)/vfnreRX) + 0.1*@sq((cR-cRX)/cRX)
```

With this revised objective function, the optimized coefficients in comparison to the original values were as follows
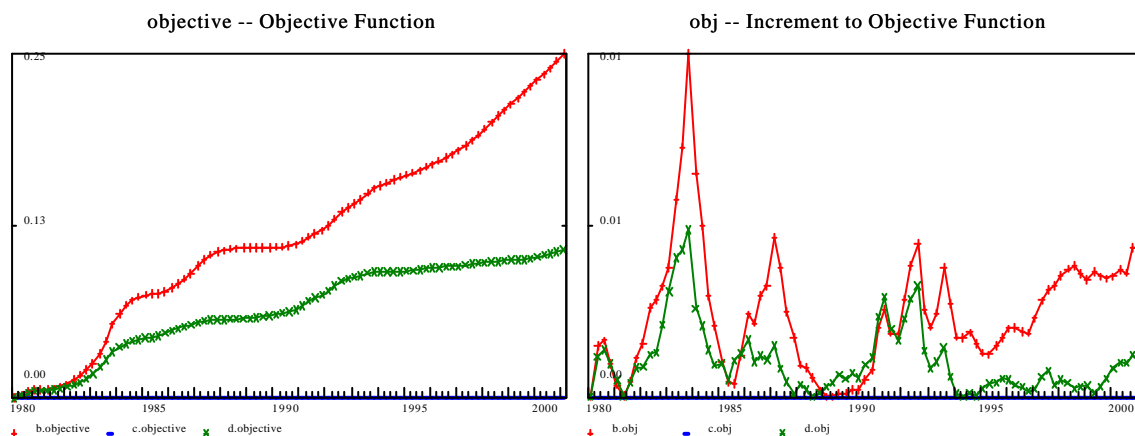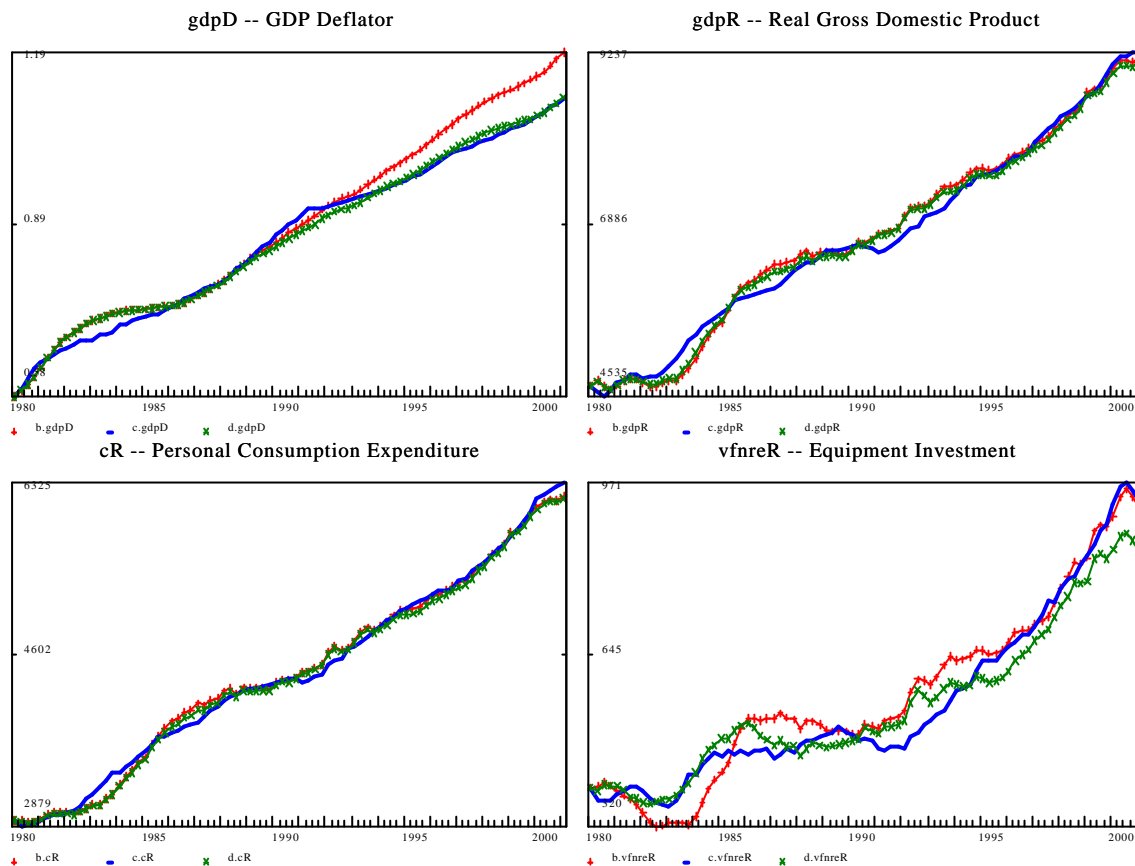
```
Resulting coeficients after optimization (108 runs).
Variable      Old:        New:
vfnreR
  intercept    36.8245    -86.2489
  ue           -4.6046      9.5125
cRpc
  intercept   785.4286    797.5327
  yRpc          0.7758      0.7600
  intsavRpc    -0.4875     -0.3995
  ur        -1416.29     -767.88
```

With this objective function, the change in the equipment investment equation is more substantial, and its unemployment term takes on a stabilizing role. In the consumption equation, on the contrary, the stabilizing role of the *ur* is reduced. The coefficient on income, where we were concerned about simultaneous equation bias, is little changed from the least-squares estimate. The reduction in the coefficient on *intsavRpc* also reduces the stabilizing effect of this variable.

As before with the simpler objective function, we get a substantial reduction in the objective function, in this case, 57 percent. Again, the biggest improvement is in the GDP deflator, where we achieve essentially a perfect simulation over the last eight years. The equipment investment simulation, as hoped, is much improved, though the performance in the last few years is not quite as good as in the model before optimization. Its weight in the objective function should perhaps be increased. All in all, however, the optimization appears to have fixed the most striking problem with the original Quest, namely, the upward creep of the GDP deflator.

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gdpD -- GDP Deflator

gdpR -- Real Gross Domestic Product

cR -- Personal Consumption Expenditure

vfnreR -- Equipment Investment

Using the optimized model

How can one use the optimized model for simulation or forecasting? Let us assume that you used OptSpec.opt as the name of the optimization specification file. Then the optimization created a file by the name of OptSpec.dat in the directory with the model. It is of exactly the format of the heart.dat file which is created to hold the coefficients for your model when you ran Build. All that you need do to run the optimized model is simply to give this file the name "heart.dat". You can simply type "dos" in the G command line box and then, in the DOS window which opens type

        copy heart.dat orig.dat
        copy optspec.dat heart.dat
        exit

If you now do Model | Run, the model you run will be the optimized one.

A word about step sizes

The efficiency, and indeed the success, of the optimization can depend on the step sizes. If they are taken too large, the model can be thrown into an unstable region in which it does not converge and the optimization fails. If they are chosen too small, either many iterations may be necessary to find an optimum, or, if they are really small so that there is little difference in the objective function at the different points and the points are very close together, the optimality test may be passed almost immediately and the process halted before it has really begun.   As a rule of thumb, I usually have taken the step sizes at about one percent of the parameter's initial value. If the size of your

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coefficients make you want to use step sizes below about .01, you should probably change the units of the variables so as to get bigger coefficients. Thus, you may need to experiment with step sizes and the units of variables to get the optimization to run smoothly.

2. Finding optimal policies

Let us turn now to finding optimal policies in a model. We will, of course, need a different objective function, one based not on closeness of fit to history but on achieving desirable social goals. We must also find a way to represent the policy variable as the dependent variable in a regression. Since this second matter requires a new technical wrinkle, let us deal with it first.

Representing policy variables by regression equations

We would like to be able to approximate a policy variable such as *pitfBR,* the federal income tax rate, by a piece-wise linear function of a relatively small number of constants, which will appear as regression coefficients and can be varied by our optimization process. Such a function is shown in the graph below.

pitfBR -- Federal Personal Tax Rate

Actual and Piecewise Linear Interpolation
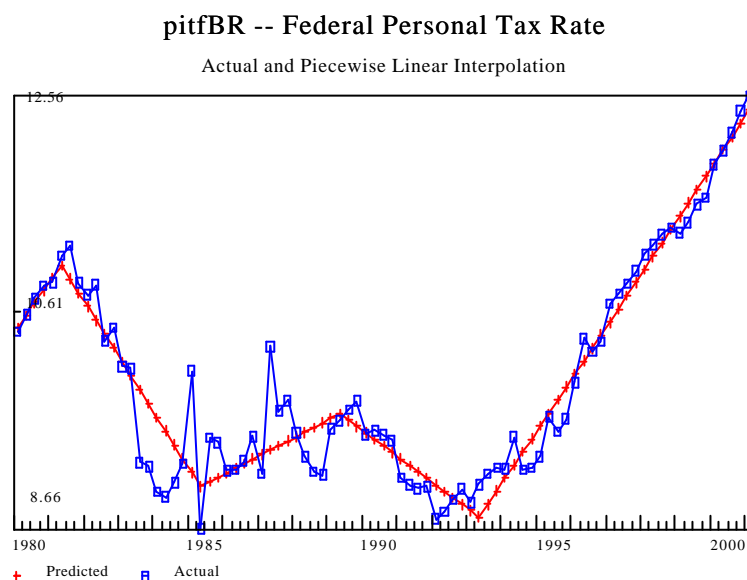
To generate the approximation by regression, we need a series of what I shall call linear interpolation functions. Each of these begins at 0 and remains 0 until its particular time interval comes; then it rises by 1 each period until the end of its interval, whereafter it remains constant at whatever value it has reached. For representing the federal personal tax rate, I took the beginning of the intervals to be the third quarter of the first year of each presidential term. Thus, except for the first which represented the tail end of the Carter policies, each of the variables rises from 0 to 16, the number of quarters in a four-year term. Here is a graph of these variables.

Interpolation Functions
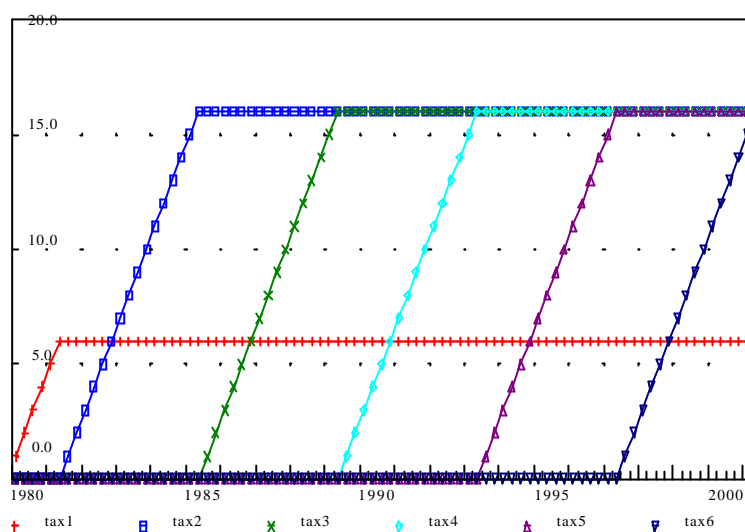
I have called these functions tax1, tax2, ..., tax6. Once we have them, we can obtain the piecewise linear approximation by a simple regression:

```
r pitfBR = tax1, tax2, tax3, tax4, tax5, tax6
```

The regression coefficients in this equation are the precisely the parameters with respect to which we optimize to find the optimal tax policy.

We could, of course, create these interpolation variables by hand and introduce them via *fex* and *update* commands into the model. G, however, offers a simpler way of generating them automatically by the *intvar* command. The command necessary to generate our six variable is

```
intvar tax 1980.1 1981.3 1985.3 1989.3 1993.3 1997.3
```

The word after the command, "tax" in this example, provides the root of the variable names which will be created by appending 1, 2, 3, etc. to this root. The dates which follow then mark the beginning of each variable's activity.

The complete regression file to compute the representation of *pitfBR* follows:

```
catch pitfBR.cat
add lim80
#  pitfBR -- Federal Personal Tax Rate

fex pTaxBase = pi - ngtpp + 0.5*nconsi + nibtax
fex pitfBR = 100.*gfrptx/pTaxBase
save pitfBR.sav
intvar tax 1980.1 1981.3 1985.3 1989.3 1993.3 1997.3
ti pitfBR -- Federal Personal Tax Rate
subti Actual and Piecewise Linear Interpolation
r pitfBR = tax1,tax2,tax3,tax4,tax5, tax6
save off
```

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```
        gname pitfBR
        gr *
        catch off
```

(The two *fex* commands above the *save* command are so placed because they are provided in the Master file.) The results of the regression are

```
:                   pitfBR -- Federal Personal Tax Rate
  SEE   =        0.27 RSQ   = 0.9250 RHO =    0.37 Obser   =    85 from 1980.100
  SEE+1 =        0.25 RBSQ  = 0.9192 DW  =    1.26 DoFree  =    78 to   2001.100
  MAPE  =        1.95
   Variable name           Reg-Coef  Mexval  Elas   NorRes     Mean    Beta
 0 pitfBR                 - - - - - - - - - - - - - - -      10.02 - - -
 1 intercept              10.35670   407.2   1.03   13.33     1.00
 2 tax1                    0.11258     3.4   0.07   13.07     5.82   0.091
 3 tax2                   -0.12431    67.4  -0.17   12.47    13.46  -0.655
 4 tax3                    0.04077    11.9   0.04   10.75    10.45   0.297
 5 tax4                   -0.05790    23.3  -0.04    9.32     7.44  -0.445
 6 tax5                    0.11661    73.5   0.05    2.24     4.42   0.790
 7 tax6                    0.11938    49.8   0.02    1.00     1.41   0.436
```

Because of the progressivity of the income tax, growth in real income increases this average tax rate. This steady upward movement during the Carter and Clinton administrations is evident in the coefficients of *tax1, tax5*, and *tax6*; the sharp cuts of the first Reagan administration shows up in the negative coefficient on *tax2*. The administration George Bush, contrary to the impression of many, cut taxes substantially, as seen in the coefficient of *tax4*.

Once this regression has been performed, it is introduced into the Master file just as any other regression with the lines

```
    #  pitax -- personal taxes and non-tax payments
    f pTaxBase = pi - ngtpp + 0.5*nconsi + nibtax
    fex pitfBR = 100.*gfrptx/pTaxBase
    # add regression for tax rate to allow optimization
    add pitfBR.sav
    id   gfrptx = .01*pitfBR*pTaxBase
```

(There is a reason for the factor of 100 in the definition of pitfBR; originally it was not there, and all the regression coefficients were 1/100 of the values shown above. The appropriate step size in the optimization therefore seemed to be about .00001. With this step size, the optimization stopped very quickly at a point very close to the initial point. In other words, it failed to optimize. Evidently, the small step size allowed the termination test to be passed long before it should have been. From this experience came the advice given above that the step sizes should not be too small.)

Putting in this additional regression meant that the optima.dat file from the optimization of the previous model no longer matched the heart.dat file for this new model. Consequently, before putting in a new objective function, I reoptimized this model with the historical fit objective function to get an Optima.dat file which could later be copied to Heart.dat so that the tax optimization should be done with the model optimized for fit. In this step, I gave at the] prompt not only the "skip

sp500R" command but also "skip pitfBR" command to use precise historical tax rates in optimizing for fit.

The specification of parameters to use in optimizing the tax rate with the socially desirable objective function was given by the following FedTax.opt file:

```
20
#Optimize tax rate
pitfBR
#   1    2    3    4    5    6     7
    1  .01  .01  .01  .01  .01  .01
```

A socially desirable objective function

Specifying a socially desirable objective function is not necessarily easy. I began with minimizing what has been called the "misery index," the sum of the unemployment rate and the unemployment rate. The optimization quickly drove unemployment negative so that $1/u$ in the consumption function became a huge negative number and the model simply broke down with attempts to take logarithms of giant or negative numbers. I then went over to the sum of the *squares* of these two misery indicators. That worked better, but took no account of the budget deficit. Paying interest on the federal debt imposes an efficiency loss in collecting the taxes with which to pay it, so I added a third misery indicator, the ratio of interest on the federal debt to GDP. Finally, to give about equal weight to all three, I took 2 percent unemployment as ideal, rather than 0 percent. The resulting objective function was then expressed by these lines in the Master file.

```
# For optimal tax
fex obj1 = 0
fex obj2 = 0
fex obj3 = 0
f obj1 = @sq(u - 2.)
f obj2 = @sq(infl)
f obj3 = @sq(100.*gfenip/gdp)
f obj = obj1+obj2+obj3
fex objective = 0
f objective = @cum(objective, obj, 0.)
```

With this modified Master, the model was then rebuilt by clicking Model | Build. *After* building, I copied the Optima.dat file to Heart.dat. Note that as long as the order of the regression equations remains unchanged, other parts of the Master file can be changed without upsetting the possibility of using the Optima.dat file in place of Heart.dat to run the model optimized for fit.

This model was then optimized with respect to the federal tax rate using the FedTax.opt file shown above. The old and new coefficients are shown below.

Changes in Federal Taxation

Variable	Historical:	Optimal
Intercept	10.3567	10.1689
Carter	0.1126	0.2090
Reagan I	-0.1243	-0.0450

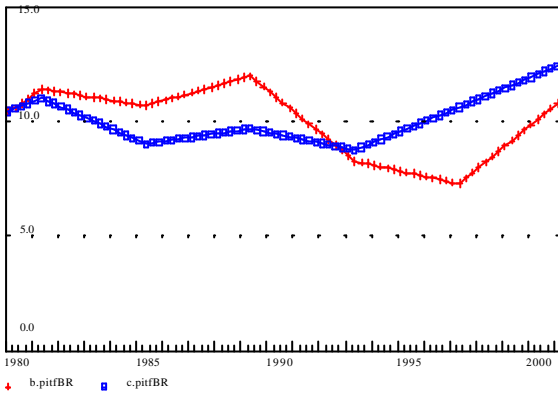|		
Reagan II	0.0408	0.0807
Bush	-0.0579	-0.2317
Clinton I	0.1166	-0.0622
Clinton II	0.1294	0.2347

The new tax rates resulting from the optimization are shown by the (red) line marked with + in the first graph below. The optimal policy would have been higher taxes in the Reagan years, a rapid drop in the Bush administration, continued low rates in the first Clinton administration, followed by a sharp rise in the second. The second graph shows that, quite unlike the objective in the optimization for fit, in this policy optimization the historical policy would have been better than the optimal one up to 1995. We seem to have a clear case of the usual macroeconomic dilemma: what is pleasant in the short run is painful in the long run and vice-versa.

The next three graphs show the effects of the tax change on the three components of the misery index we are minimizing. All three are plotted on the same scale to facilitate comparison of the contribution. The following three show these variables in the misery index in more customary units without squaring; the last two graphs show real GDP and the GDP deflator.
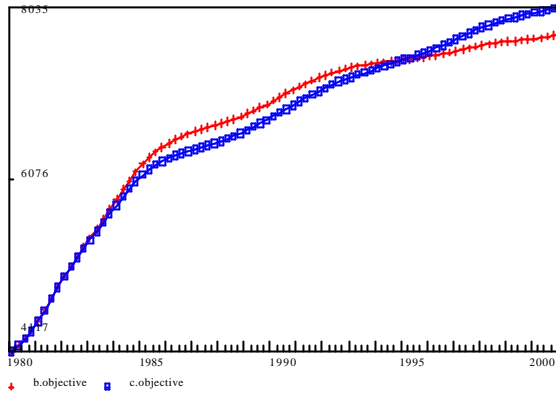
The optimal tax policy accepted a bit more unemployment and some loss in real GDP early in the simulation in order to get higher real GDP, lower unemployment, and much lower prices near the end of the period. Inflation with the optimized tax rate is lower throughout the period except for the last three years where it rises slightly. The interest component of the objective function is uniformly reduced. Though this component does not have the spike in the early 1980's that the others do, the difference between the two lines is of similar magnitude to the differences of the other two indicators.

The next three graphs show these indicators in more natural units without squaring. The final two show that the optimal tax policy would have made slight changes in real GDP but large differences in the price level.
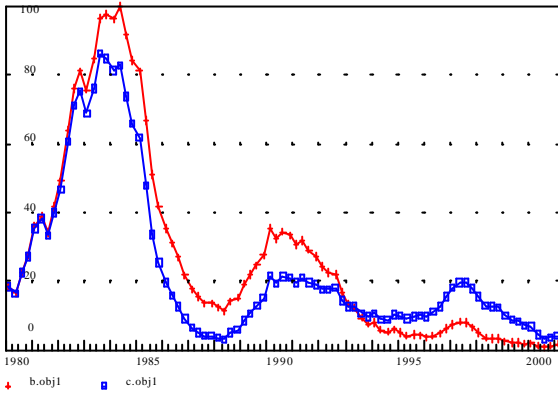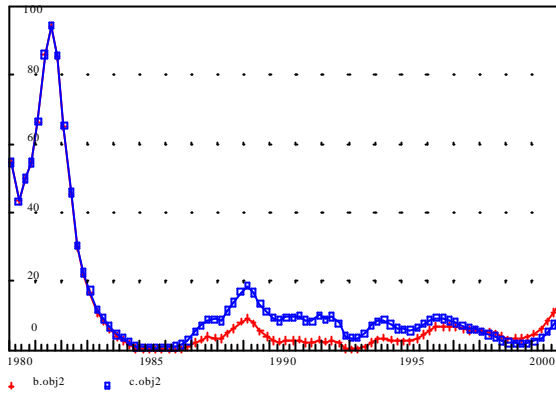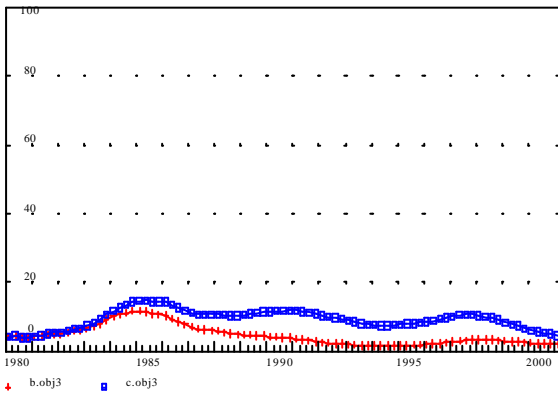
Federal Personal Tax Rate

b.pitfBR   c.pitfBR

Objective

b.objective   c.objective

Obj1 Unemployment

b.obj1   c.obj1

Obj2 Inflation

b.obj2   c.obj2

Obj3 Interest

b.obj3   c.obj3

u -- Unemployment

b.u   c.u
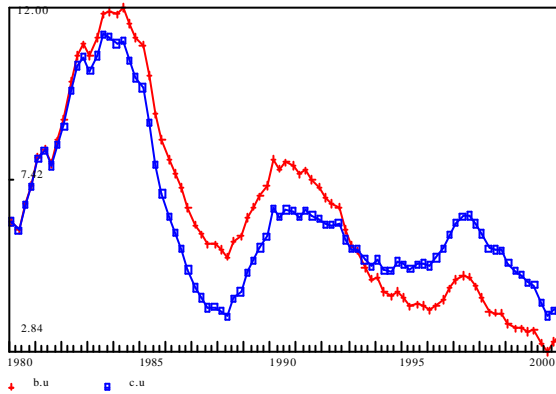
infl -- Inflation Rate of GDP Deflator

b.infl   c.infl
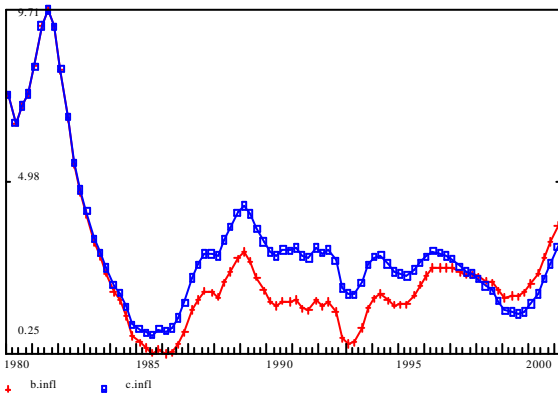
Federal Interest as Percent of GDP

bint   cint

gdpR -- Real Gross Domestic Product   gdpD -- GDP Deflator
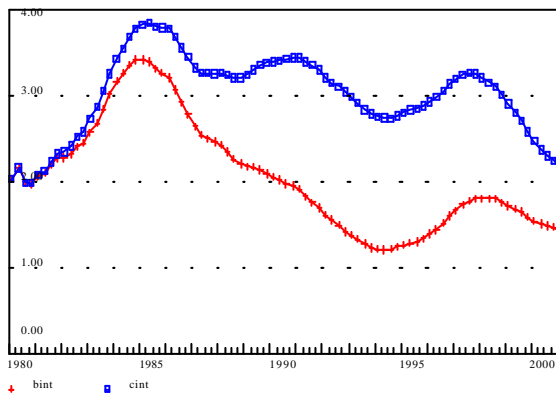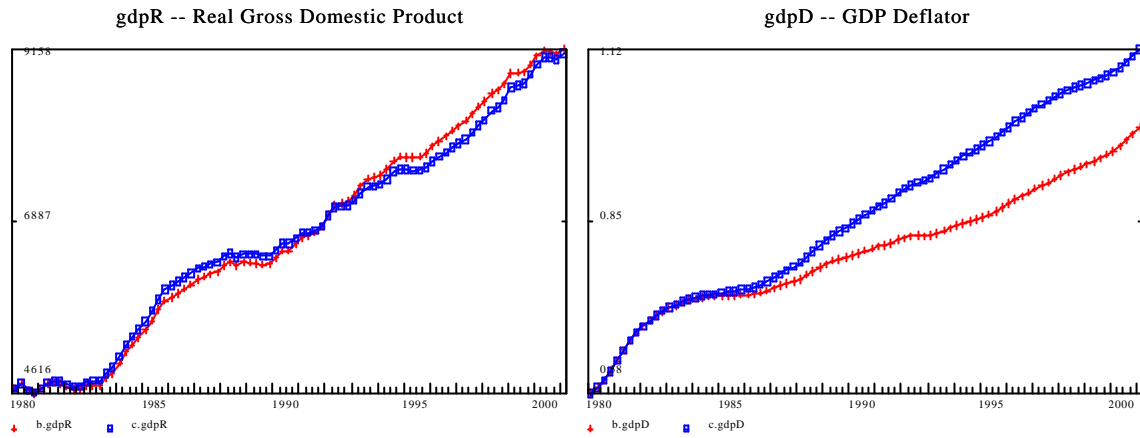
Though these results bear out my own beliefs that the Reagan tax cuts were utterly irresponsible, different objective functions would give different optimal policies. The exercise does, however, illustrate how models can be used in designing of policies.